EGSIEM

European Gravity Service for Improved Emergency Management

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Foreword

These are not traditional Power Point slides. Why? Well, as all who were there will remember, this lecture didn't have any Power Point slides. None. It was a *good-old-fashioned chalk-and-blackboard* lecture. Why so? I love this old style of teaching, and I find it more interactive than any other. Students also remember that I just kept coming up with problems and questions, asking them how to solve them. It is always a pleasure for me to see the students' eyes light up to seek solutions before I give answers, and to actively grasp essential concepts. Experience proves that the rate of remembrance is far higher when people participate than when they just sit and listen. And the feedback is generally good. So what did we talk about? These are a few retrospective slides that will serve as a reminder for all those who were present.

Summary

So, if you remember well, the lecture was in three parts:

- An introduction to gravity field. Why do we monitor Earth's gravity field? What are spherical harmonics? What is orbit restitution? And finally, the dream of every scientist: minimize the residuals.
- The second part was entitled « Least Squares Revisited ». That was the mathematical part, on the topic of singular value decomposition, which was specifically intended to give you headaches (although it didn't succeed). In visual terms, if you're standing somewhere on a mountain slope, what's the fastest direction to reach the bottom of the valley? Or, well, is it actually worth to go that low?
- The third part was designed to make you feel better after all that heavy theory: discover a great interactive tool for scientists: « The EGSIEM Plotter », and its multiple possibilities.

Introduction to gravity field (1/2)

Let's remember the key points of the lecture:

- Gravity is due to the presence of mass. When mass changes or moves around, the gravity field changes. Since water represents most of the moving mass on Earth, by monitoring Earth's gravity field from space, we can have access to the movements of water at its surface: continental hydrology, droughts, floods, monsoons, ice melt. The GRACE gravity mission is a very powerful tool with which to monitor climate and water on Earth.
- Next point: we talked about the geoid, which is a gravity equipotential, and about different ways to express gravity field: geoid heights and equivalent water heights (equivalent layers of water at the surface of the Earth).
- We also talked about spherical harmonics, or how to accurately represent a potato shape using an infinite series of mathematical functions. Some sort of cousin of the Fourier series, only in 3 dimensions.

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Introduction to gravity field (2/2)

- We then talked about orbit restitution, measurements, models, theoretical quantities, measured quantities, and the difference between both: residuals.
- If your models are perfect and your instruments are perfect, you will be able to numerically generate an orbit that has the exact same characteristics as your measurements. Your residuals will be zero.
- Of course, this never happens, but it will forever remain the holy grail of geodesists: to make such an accurate and precise modelization that your residuals are as close to zero as possible.
- We then switched gears to the Least Squares Minimization.

Least Squares Revisited (1/3)

In this part, we did not talk about the classic « Highway 61 Revisited » by Bob Dylan, but rather about « Least Squares Revisited ». But they are at least as poetical for the mathematical mind.

- So, what is the least square function? Let's remember. It is the sum of the squares of the residuals in function of the parameters. On the vertical z axis is the value of the residuals, and on the horizontal plane are the parameters (x in one dimension, or (x,y) in two dimensions, or (x1,…,xn) in n dimensions).
- What does this function look like? It's quadratic. In one dimension, it is a parabola. In two dimensions, it is a paraboloïd. In *n* dimensions, it is a hyper-paraboloïd. In all cases, it is from the same family as a parabola and it has a minimum that's easy to find.

Least Squares Revisited (2/3)

- The minimum of this function provides the smallest level of residuals. It represents your ideal parameter according to the least squares criterion. The minimum of the paraboloïd is the solution to your problem.
- Suppose you are sitting somewhere on the paraboloïd side, and looking for your solution. Your goal is to slide down to the minimum of the paraboloïd, a.k.a the very bottom of the « valley ». Now look: from where you are, the slope might be different if you are heading in the West direction or in the South direction. If the slope is sharp, it will be a very fast and efficient slide down: you will lower your altitude a lot while not moving a lot in the horizontal plane. On the contrary, if the slope is flat, the story is different: you can find yourself travelling for very long in the horizontal plane and still not lower your altitude by very much.

Least Squares Revisited (3/3)

- So, the first thing you want to do is make efficient moves, which will minimize your altitude by traveling along the directions where the slope is very sharp. You might not want to travel for long in very flat directions for a very small gain in altitude. So, move only along directions with sharp slopes, and stay still along flat slopes. Replace the flat slope parabola by a line, and replace the paraboloïd by a cylinder. That's called Truncation of Eigen Values. All this has a mathematical formulation, but it is not important for the course of this lecture.
- What is interesting is the effect on gravity field restitution: instead of considering the minimum of the paraboloïd as our solution, we consider a partial minimization only along important directions. You can see the results in the EGSIEM blog, in the articles « On the importance of focusing efforts in the right directions », Part 1 & 2. It is a very efficient way to remove striping in the GRACE gravity field solutions.

The EGSIEM Plotter

So we now come to part 3 of the lecture. After suffering through the theoretical explanations on gravity field and mathematics, the third part of the class was designed to let you play around with our unique tool at EGSIEM: the EGSIEM Plotter. You can visualize the gravity field evolution on any point or area of the map with just a few clicks of a mouse. The possibilities are endless, and the best way to discover this now is to try it yourself at plot.egsiem.eu.

Conclusion

That was a short introduction to what we do in gravity field restitution in the EGSIEM project. The application of Singular Value Decomposition to GRACE gravity field is a specific work we did in Toulouse between Géode & Cie and CNES, and an alternative to the traditional filtering of solutions. This reminds us that it is always good to have several different groups working on a same subject, as it brings diversity and different approaches. And then projects like EGSIEM that allows us to collaborate and share our views on the subject. We hope that you enjoyed the class, and that a few of you will catch the space geodesy bug and join us in the gravity bandwagon in the future.

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