GRACE Analysis

Torsten Mayer-Gürr

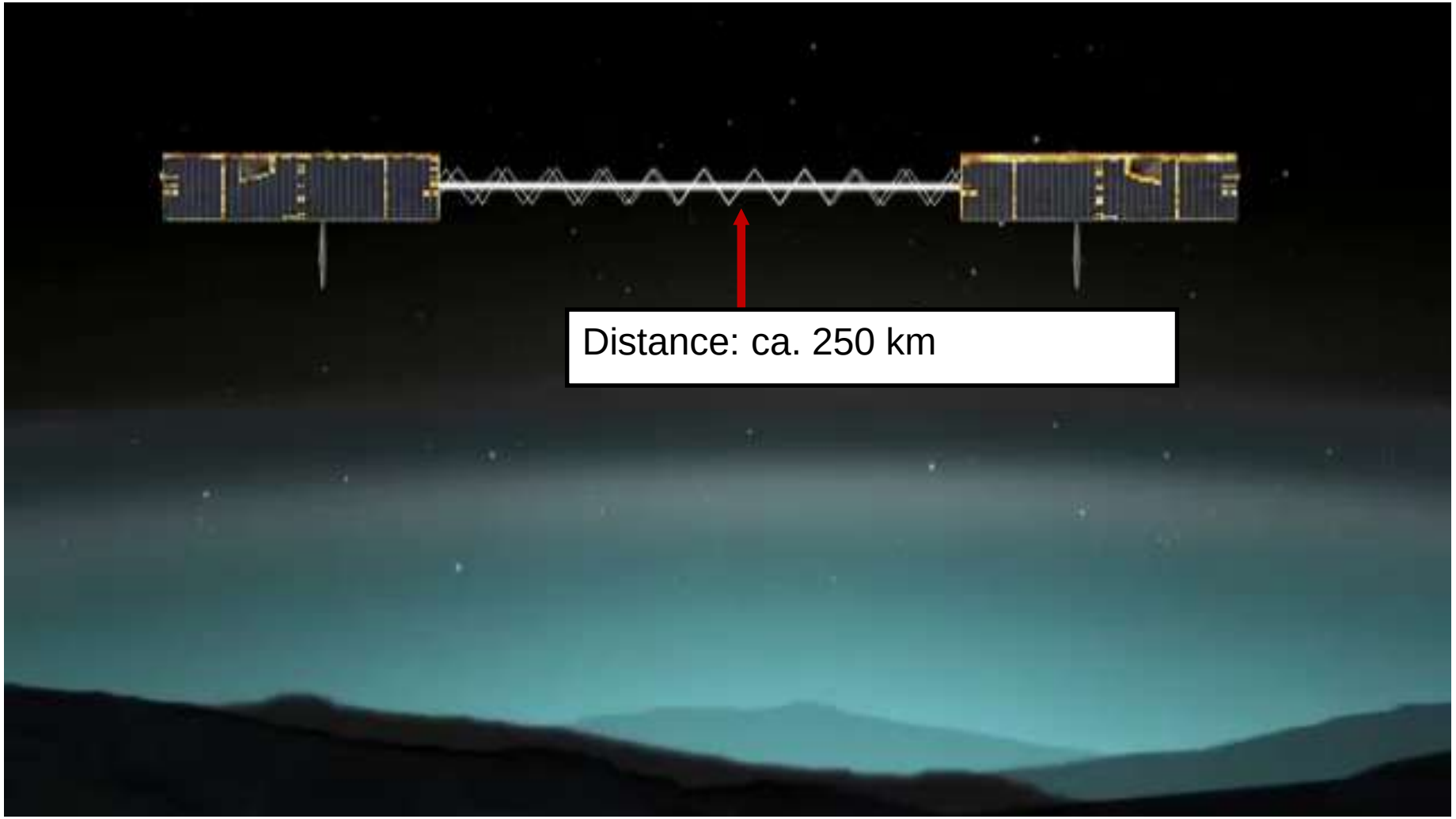
Institute of Geodesy
Graz University of Technology

EGSIEM autumn school

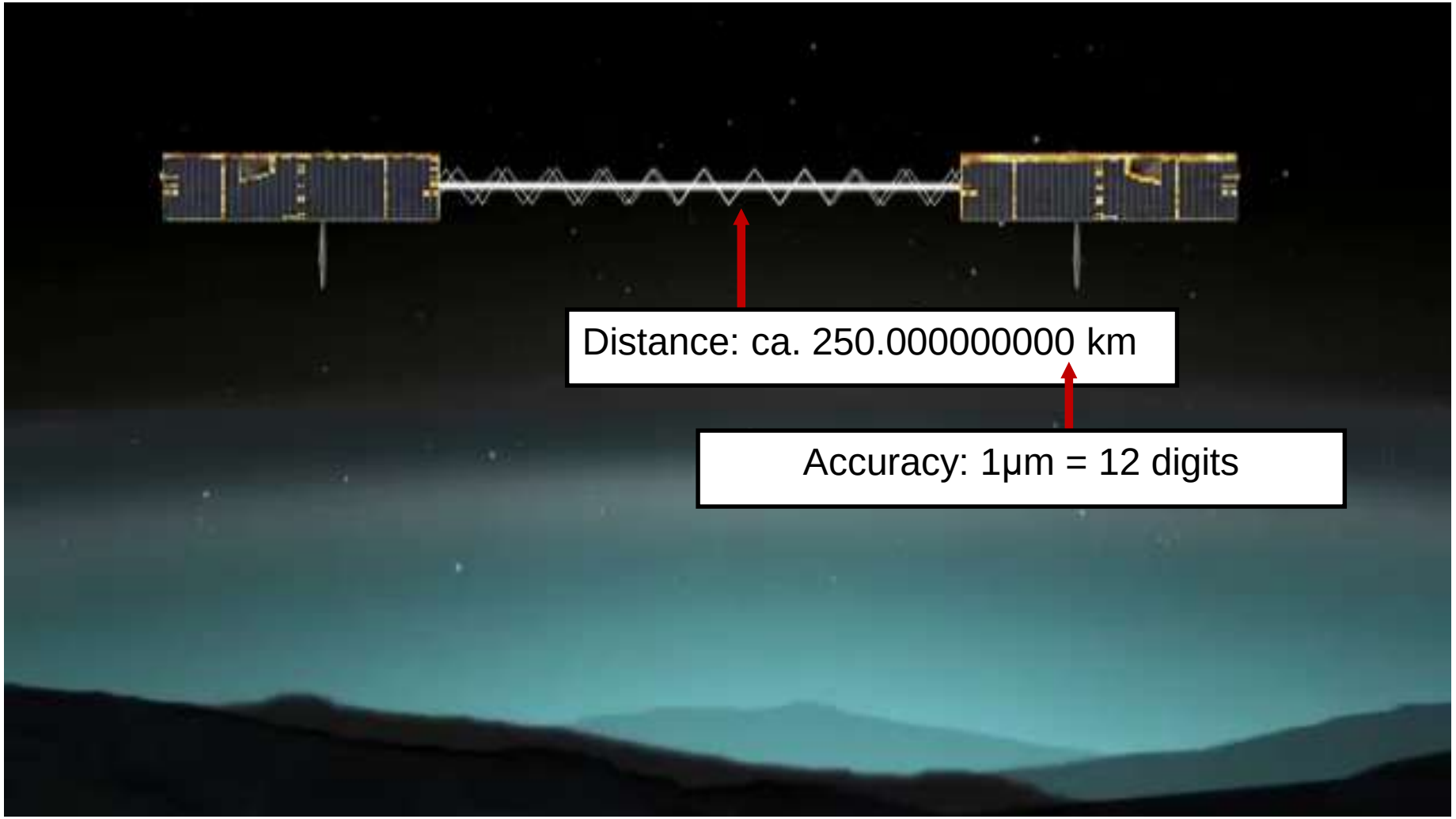
2017-09-12

Gravity Recovery and Climate Experiment

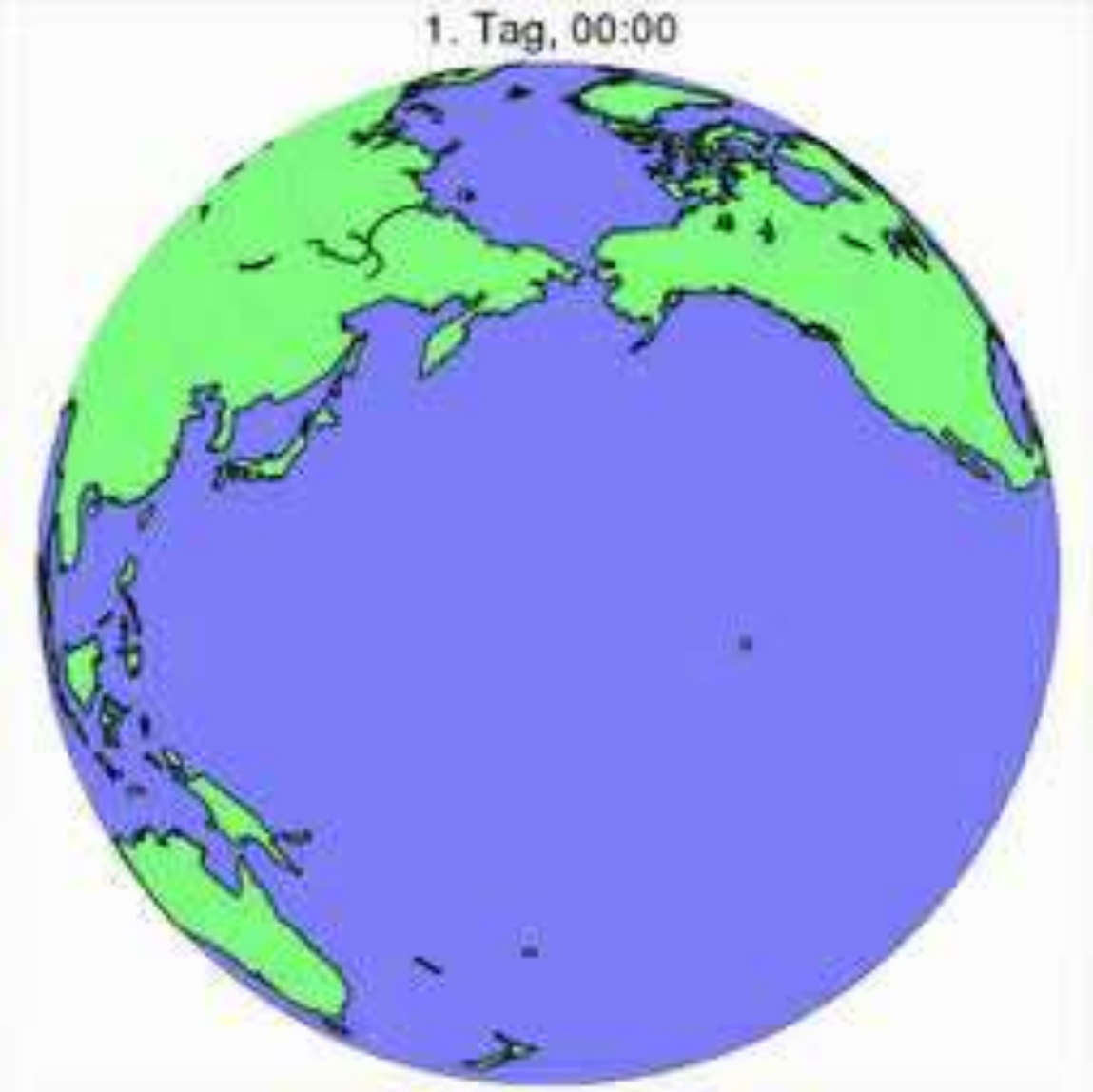
Gravity Recovery and Climate Experiment



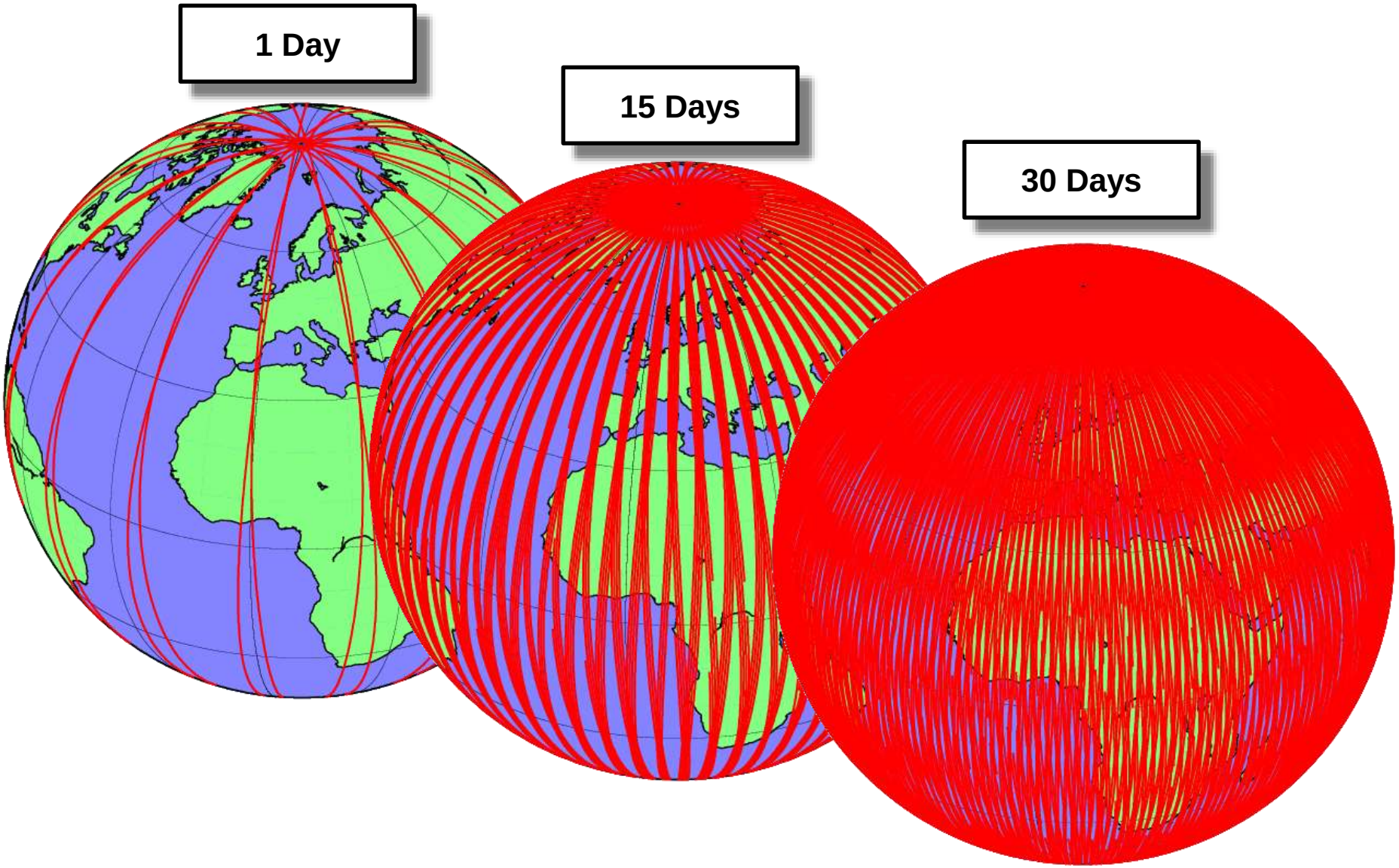
Gravity Recovery and Climate Experiment



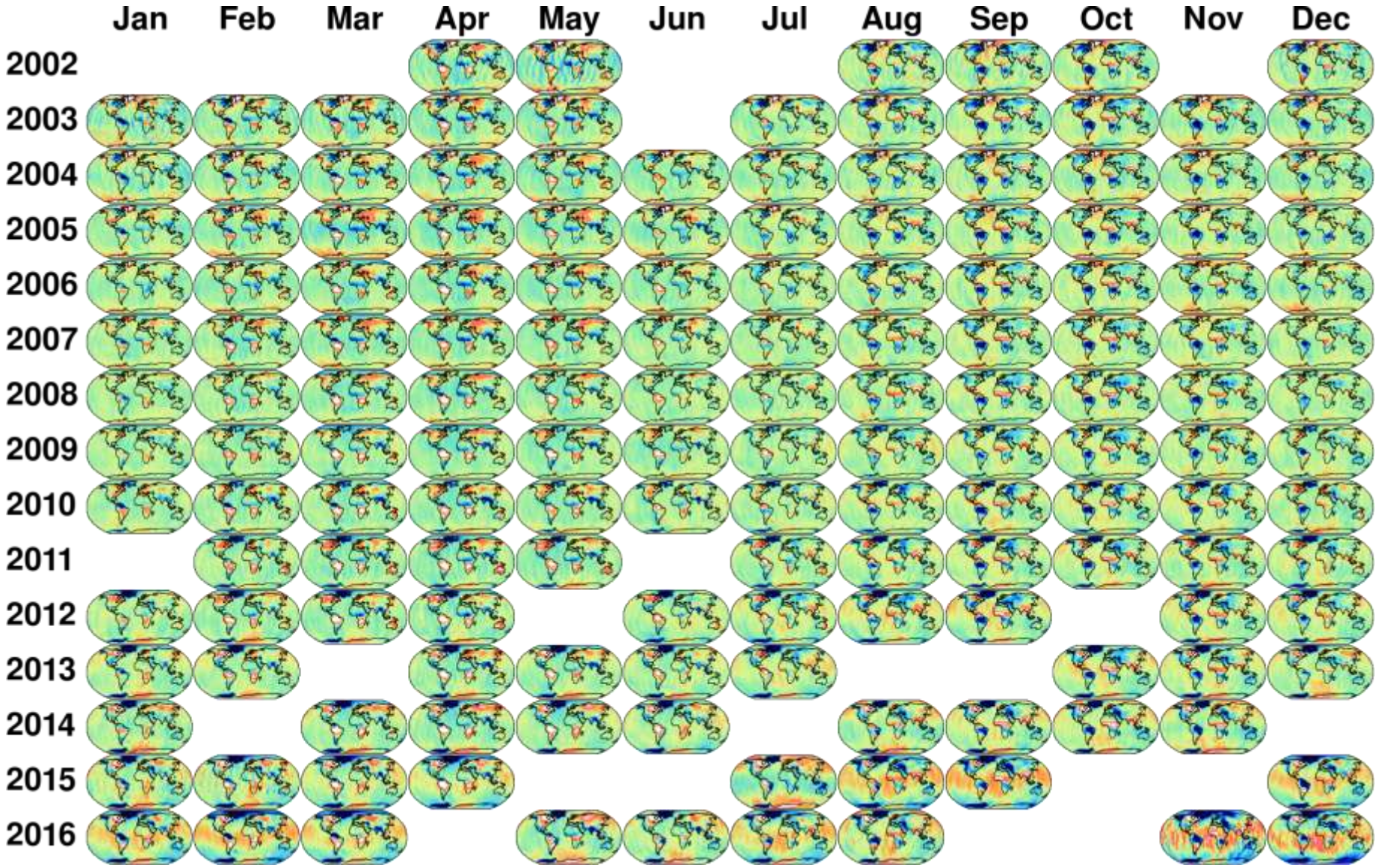
Groundtracks



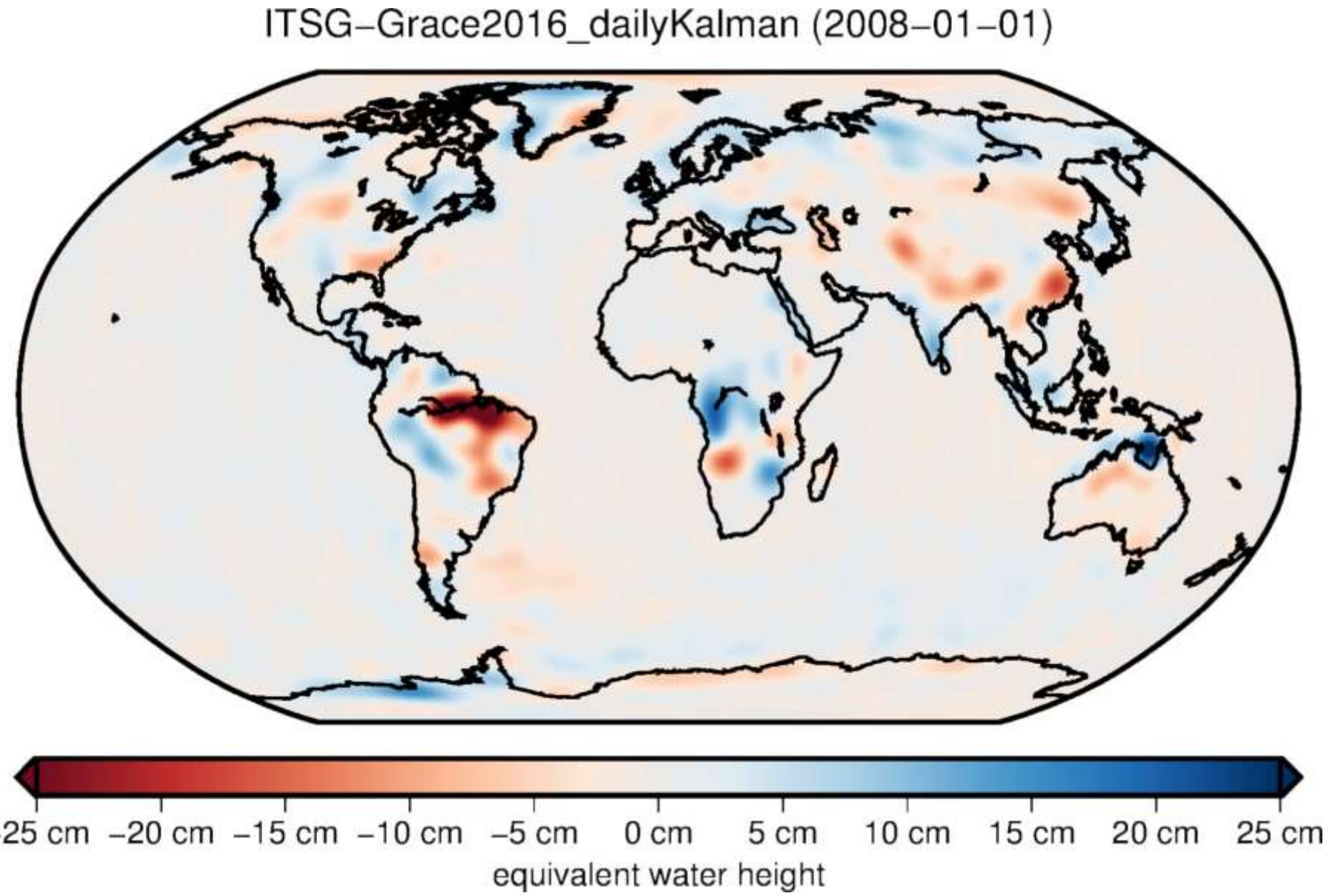
Groundtracks



GRACE Monthly solutions



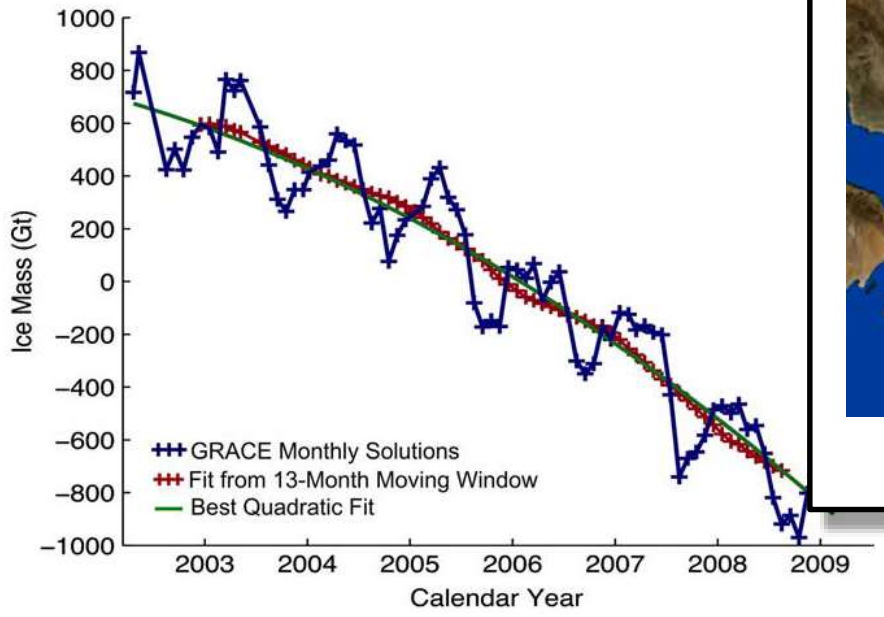
ITSG-Grace2016 daily (30 day smoothing)



Gravity Recovery and Cli

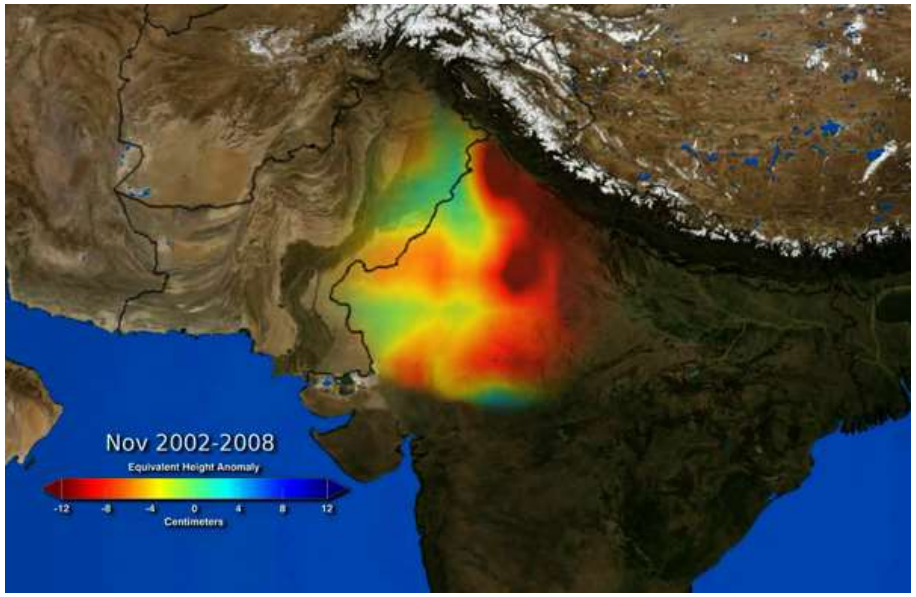
Jan Feb Mar Apr May

Ice mass variations in Greenland



Velicogna, 2009, Geophys. Res. Lett.

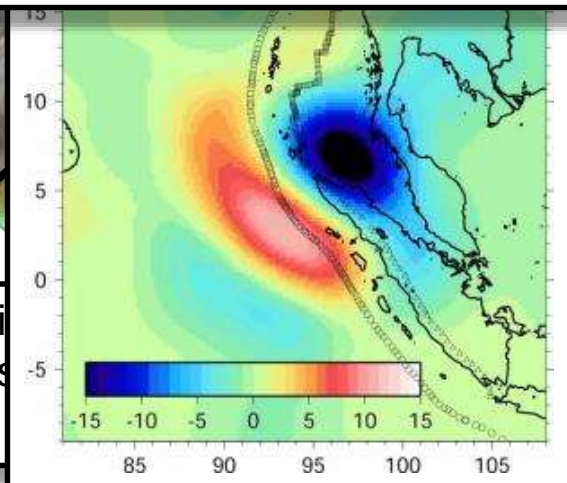
Groundwater depletion in North India



Rodell et al., 2009, Nature

2013
2014
2015
2016

More than 3.000 Publicati
(GRACE Tellus, grace.jpl.nas
from March 2017)



Han et al., 2006, Science

Gravity Recovery and Clim

Jan Feb Mar Apr May

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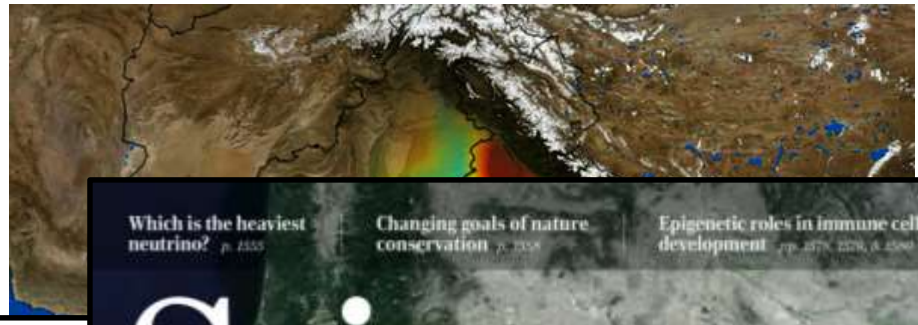
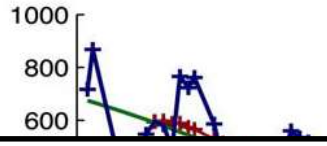
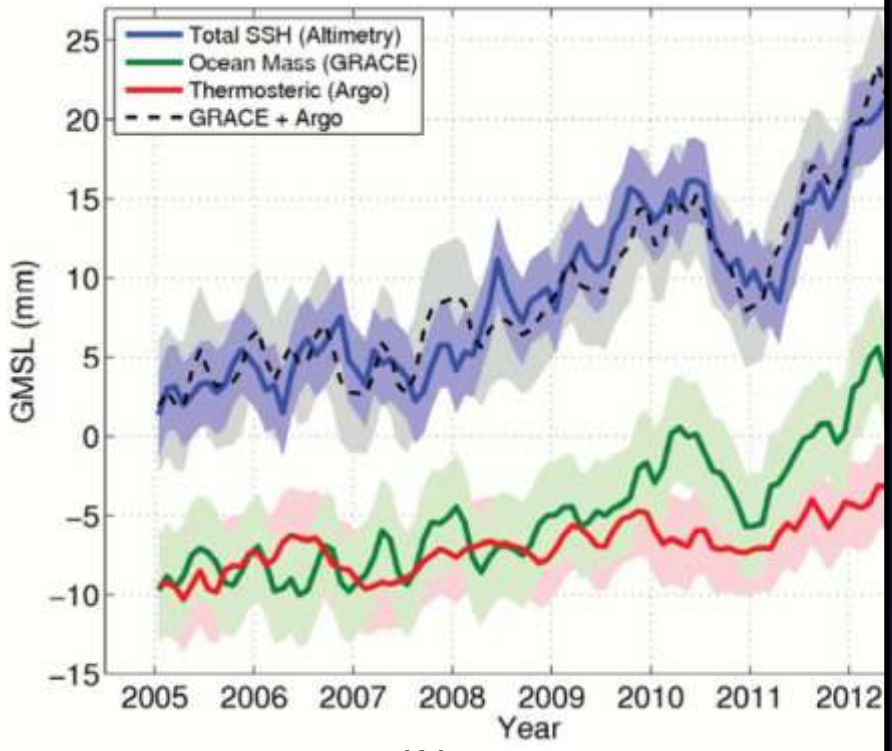


Ice mass variations in Greenland



Global Sea level change



IPCC WGI Fifth Assessment Report, 2

Data bases

Data base

- **General information**
  - GRACE tellus: grace.jpl.nasa.gov
    - Information
    - Publications
    - Data
  - International Centre for Global Earth Models (ICGEM) icgem.gfz-potsdam.de/ICGEM/
  - EGSIEM egsiem.eu

- **Processing centers**
  - GFZ isdc.gfz-potsdam.de
  - CSR csr.utexas.edu/grace
  - JPL podaac.jpl.nasa.gov/gravity/grace
  - CNES: grgs.obs-mip.fr/grace
  - Uni Bern aiub.unibe.ch
  - TU Graz ifg.tugraz.at
  - Tongji University
  - Wuhan University
  - TU Delft
  - …

- **Combination**
  - EGSIEM

ICGEM


Introduction
ICGEM International Center for
ICGEM International Center for
icgem.gfz-potsdam.de/series
Suchen
ICGEM Home
Gravity Field Models
Static Models
Temporal Models
Topography related Models
Calculation Service
3D Visualisation
Static Models
Temporal Models
Trend & Amplitude
Spherical Harmonics
Evaluation
Spectral domain
GNSS Leveling
FAQ
IAG
I C G E M
GFZ
Helmholtz Centre
POTSDAM
Gravity Field Solutions for dedicated Time Periods
The following gravity field time series are presently available:
GRACE monthly solutions from the 3 processing centers CSR, GFZ and JPL
CSR Release 05 (UTCSR Level-2 Processing Standards Document, Rev 4.0 May 29, 2012)
GFZ Release 05 (GFZ GRACE Level-2 Processing, Revised Edition, January 2013)
JPL Release 05 (PL Level-2 Processing Standards Document, Release 05.1 November 3, 2014)
The processing standards to generate the GRACE Level-2 products of CSR, GFZ and JPL
are also available in the Document Section of the GRACE archives at GFZ ISDC or JPL PO.DAAC
GRACE / CHAMP monthly solutions from other groups
AIUB Release 02 (more information can be found here)
CNES_GRGS (GRACE solutions release 03; more information can be found here)
DMT-1 (more information can be found here)
EGSIEM (GRACE monthly combined solutions from the EGSIEM project, more information can be found here)
HUST-Grace2016 DOI (GRACE monthly solutions from the Huazhong University of Science and Technology, Wuhan, PR China)
ITG (more information can be found here)
ITSG-Grace2014 (GRACE monthly solutions from the ITSG, TU Graz; more information can be found here)

Gravity field solution

```
product_type            gravity_field
modelname               ITG-Grace03
comment                 static field from 2002-08 to 2007-04 of GRACE data
earth_gravity_constant 3.986004415e+14
radius                  6378136.6
max_degree              180
key n m        C                    S                    sigma C              sigma S
end_of_head ===================================================================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 1  0.000000000000e                                          +00 0.000000000000e+00
gfc 2 0 -4.841692718699e                                          -13 0.000000000000e+00
gfc 2 1 -2.654790999243e                                          -13 6.355307212507e-13
gfc 2 2  2.439383367978e                                          -13 6.423410956098e-13
gfc 3 0  9.571610348416e                                          -13 0.000000000000e+00
gfc 3 1  2.030461736678e                                          -13 5.118675157415e-13
gfc 3 2  9.047877724984e                                          -13 5.482674767117e-13
gfc 3 3  7.213217237276e                                          -13 5.163483433061e-13
gfc 4 0  5.399657665980e                                          -13 0.000000000000e+00
gfc 4 1 -5.361573220519e                                          -13 3.973550735355e-13
gfc 4 2  3.505015650151e                                          -13 4.398486563277e-13
gfc 4 3  9.908565738322e                                          -13 4.766590461153e-13
gfc 4 4 -1.885196275153e                                          -13 4.565287154679e-13
gfc 5 0  6.867029195170e-08  0.000000000000e+00 2.444626602968e-13 0.000000000000e+00
gfc 5 1 -6.292117216708e-08 -9.436975416042e-08 2.510438328896e-13 2.640391465060e-13
...
```

Spherical harmonics ?

Where is the water storage?

Outline

- Approximation of functions on the sphere
  - Spherical harmonics
- Gravity field
  - Upward continuation
  - Orders of magnitude
  - Functionals of the gravity field
- Accuracy of gravity fields
  - Degree variances
- Filtering
- Signal content
  - Physical interpretation of the coefficients
- Evaluation of GRACE solutions: Step by Step

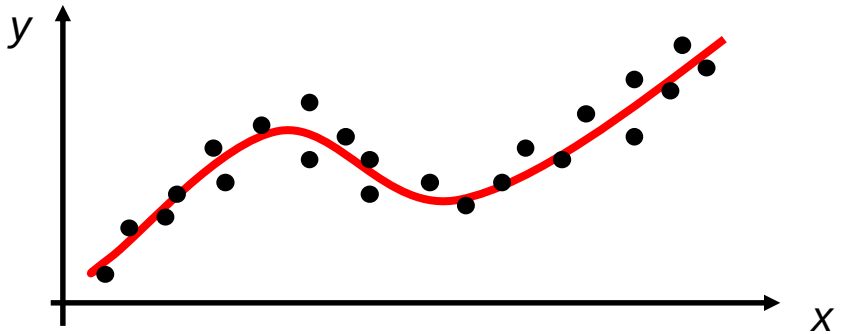
Approximation of functions on the sphere

Approximation

Approximation with a polynomial of degree n:

$$f(x) = a_0 p_0(x) + a_1 p_1(x) + \cdots + a_n p_n(x)$$

$$p_n(x) = x^n$$



Approximation of a periodic function with a Fourier series:

$$f(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos\left(m \frac{2\pi}{T} t\right) + s_n \sin\left(m \frac{2\pi}{T} t\right)$$

Approximation of a function on the sphere with spherical harmonics:

$$f(\lambda, \vartheta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm} Y_{nm}(\lambda, \vartheta)$$

basis functions

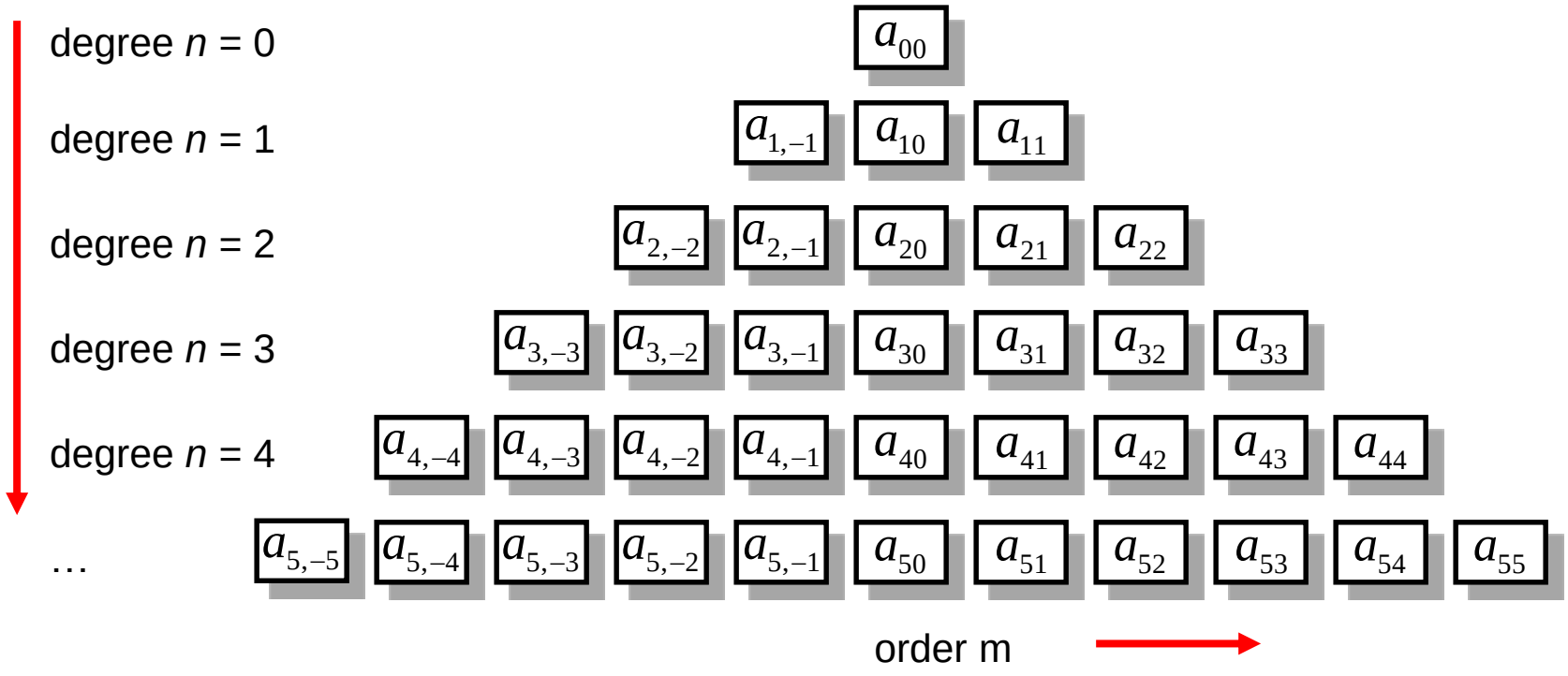
degree *n* order *m* coefficients (scale)

Coefficient triangle

Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}Y_{nm}(\lambda,\vartheta)$$

Arrangement of coefficients in a triangle



Gravity field solution

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product_type            gravity_field
modelname               ITG-Grace03
comment                 static field from 2002-08 to 2007-04 of GRACE data
earth_gravity_constant  3.986004415e+14
radius                  6378136.6
max_degree              180
```

degree

			C	S	sigma C	sigma S
gfc	0	0	1.000000000000e+00	0.000000000000e+00	0.000000000000e+00	0.000000000000e+00
gfc	1	0	0.000000000000e+00	0.000000000000e+00	0.000000000000e+00	0.000000000000e+00
gfc	1	1	0.000000000000e+00	0.000000000000e+00	0.000000000000e+00	0.000000000000e+00
gfc	2	0	-4.841692718699e-04	0.000000000000e+00	6.469883774458e-13	0.000000000000e+00
gfc	2	1	-2.654790999243e-10	1.475393314283e-09	6.108979511966e-13	6.355307212507e-13
gfc	2	2	2.439383367978e-06	-1.400273635220e-06	6.254221806143e-13	6.423410956098e-13
gfc	3	0	9.571610348416e-07	0.000000000000e+00	4.908157850872e-13	0.000000000000e+00
gfc	3	1	2.030461736678e-06	2.482003394707e-07	4.904543816334e-13	5.118675157415e-13
gfc	3	2	9.047877724984e-07	-6.190053685183e-07	5.459595906001e-13	5.482674767117e-13
gfc	3	3	7.213217237276e-07	1.414349090196e-06	5.163836126113e-13	5.163483433061e-13
gfc	4	0	5.399657665980e-07	0.000000000000e+00	3.758481731782e-13	0.000000000000e+00
gfc	4	1	-5.361573220519e-07	-4.735672404588e-07	3.874699557956e-13	3.973550735355e-13
gfc	4	2	3.505015650151e-07	6.624798955603e-07	4.501829959710e-13	4.398486563277e-13
gfc	4	3	9.908565738322e-07	-2.009566568843e-07	4.776067657084e-13	4.766590461153e-13
gfc	4	4	-1.885196275153e-07	3.088038091544e-07	4.556511148108e-13	4.565287154679e-13
gfc	5	0	6.867029195170e-08	0.000000000000e+00	2.444626602968e-13	0.000000000000e+00
gfc	5	1	-6.292117216708e-08	-9.436975416042e-08	2.510438328896e-13	2.640391465060e-13

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radius                   6378136.6
max_degree               180
k        order     C                  S                   sigma C             sigma S
en       ====================================================================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 1  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 2 0 -4.841692718699e-04  0.000000000000e+00 6.469883774458e-13 0.000000000000e+00
gfc 2 1 -2.654790999243e-10  1.475393314283e-09 6.108979511966e-13 6.355307212507e-13
gfc 2 2  2.439383367978e-06 -1.400273635220e-06 6.254221806143e-13 6.423410956098e-13
gfc 3 0  9.571610348416e-07  0.000000000000e+00 4.908157850872e-13 0.000000000000e+00
gfc 3 1  2.030461736678e-06  2.482003394707e-07 4.904543816334e-13 5.118675157415e-13
gfc 3 2  9.047877724984e-07 -6.190053685183e-07 5.459595906001e-13 5.482674767117e-13
gfc 3 3  7.213217237276e-07  1.414349090196e-06 5.163836126113e-13 5.163483433061e-13
gfc 4 0  5.399657665980e-07  0.000000000000e+00 3.758481731782e-13 0.000000000000e+00
gfc 4 1 -5.361573220519e-07 -4.735672404588e-07 3.874699557956e-13 3.973550735355e-13
gfc 4 2  3.505015650151e-07  6.624798955603e-07 4.501829959710e-13 4.398486563277e-13
gfc 4 3  9.908565738322e-07 -2.009566568843e-07 4.776067657084e-13 4.766590461153e-13
gfc 4 4 -1.885196275153e-07  3.088038091544e-07 4.556511148108e-13 4.565287154679e-13
gfc 5 0  6.867029195170e-08  0.000000000000e+00 2.444626602968e-13 0.000000000000e+00
gfc 5 1 -6.292117216708e-08 -9.436975416042e-08 2.510438328896e-13 2.640391465060e-13
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radius                   6378136.6
max_degree               180
key n m                                                     sigma C              sigma S
end_of_he                                                   ====================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 1  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 2 0 -4.841692718699e-04  0.000000000000e+00 6.469883774458e-13 0.000000000000e+00
gfc 2 1 -2.654790999243e-10  1.475393314283e-09 6.108979511966e-13 6.355307212507e-13
gfc 2 2  2.439383367978e-06 -1.400273635220e-06 6.254221806143e-13 6.423410956098e-13
gfc 3 0  9.571610348416e-07  0.000000000000e+00 4.908157850872e-13 0.000000000000e+00
gfc 3 1  2.030461736678e-06  2.482003394707e-07 4.904543816334e-13 5.118675157415e-13
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gfc 4 0  5.399657665980e-07  0.000000000000e+00 3.758481731782e-13 0.000000000000e+00
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gfc 4 2  3.505015650151e-07  6.624798955603e-07 4.501829959710e-13 4.398486563277e-13
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gfc 5 1 -6.292117216708e-08 -9.436975416042e-08 2.510438328896e-13 2.640391465060e-13
...
```

coefficients

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radius                  6378136.6
max_degree              180
key n m        C                    S                    accuracies
end_of_head ===================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 1  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 2 0 -4.841692718699e-04  0.000000000000e+00 6.469883774458e-13 0.000000000000e+00
gfc 2 1 -2.654790999243e-10  1.475393314283e-09 6.108979511966e-13 6.355307212507e-13
gfc 2 2  2.439383367978e-06 -1.400273635220e-06 6.254221806143e-13 6.423410956098e-13
gfc 3 0  9.571610348416e-07  0.000000000000e+00 4.908157850872e-13 0.000000000000e+00
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gfc 3 3  7.213217237276e-07  1.414349090196e-06 5.163836126113e-13 5.163483433061e-13
gfc 4 0  5.399657665980e-07  0.000000000000e+00 3.758481731782e-13 0.000000000000e+00
gfc 4 1 -5.361573220519e-07 -4.735672404588e-07 3.874699557956e-13 3.973550735355e-13
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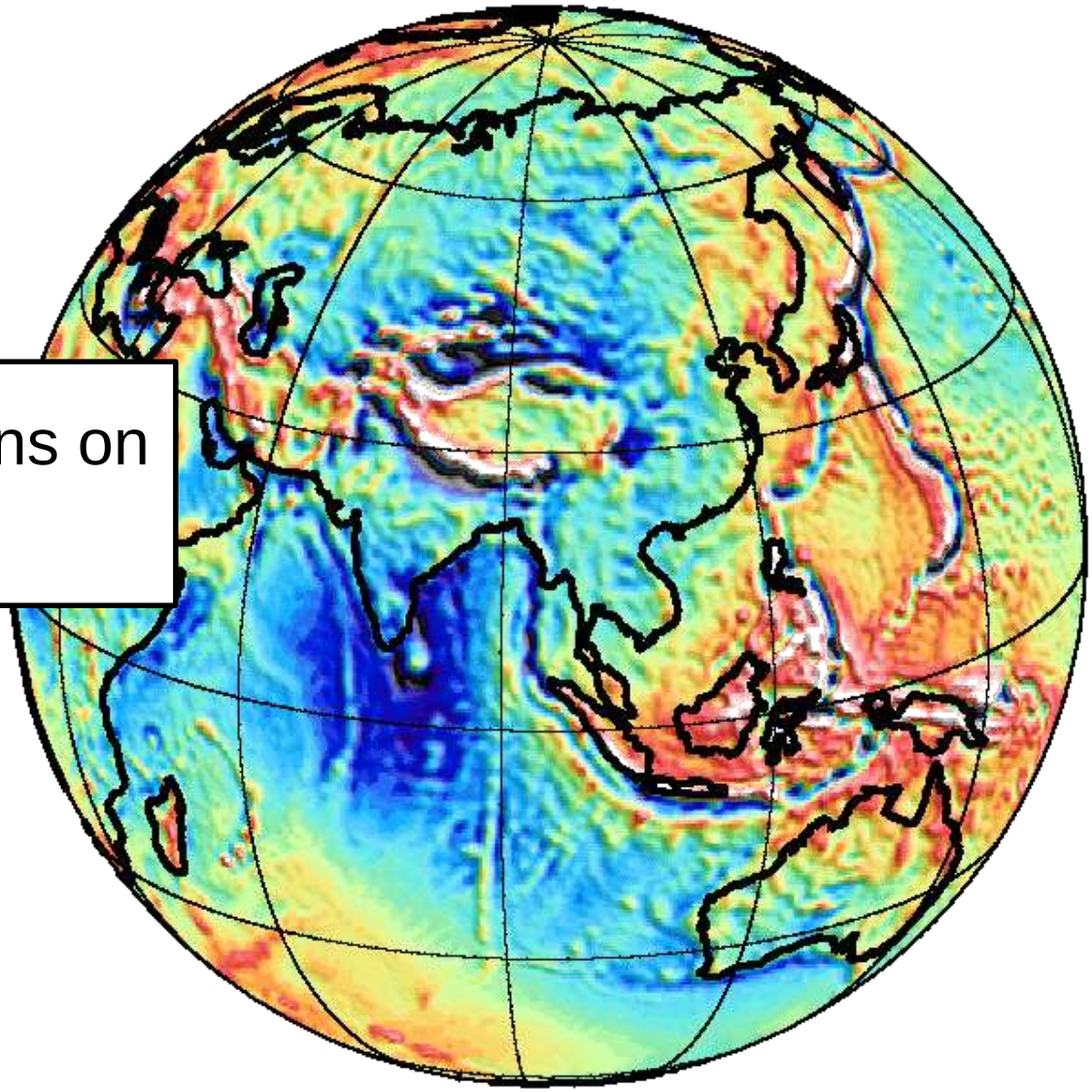
...
```

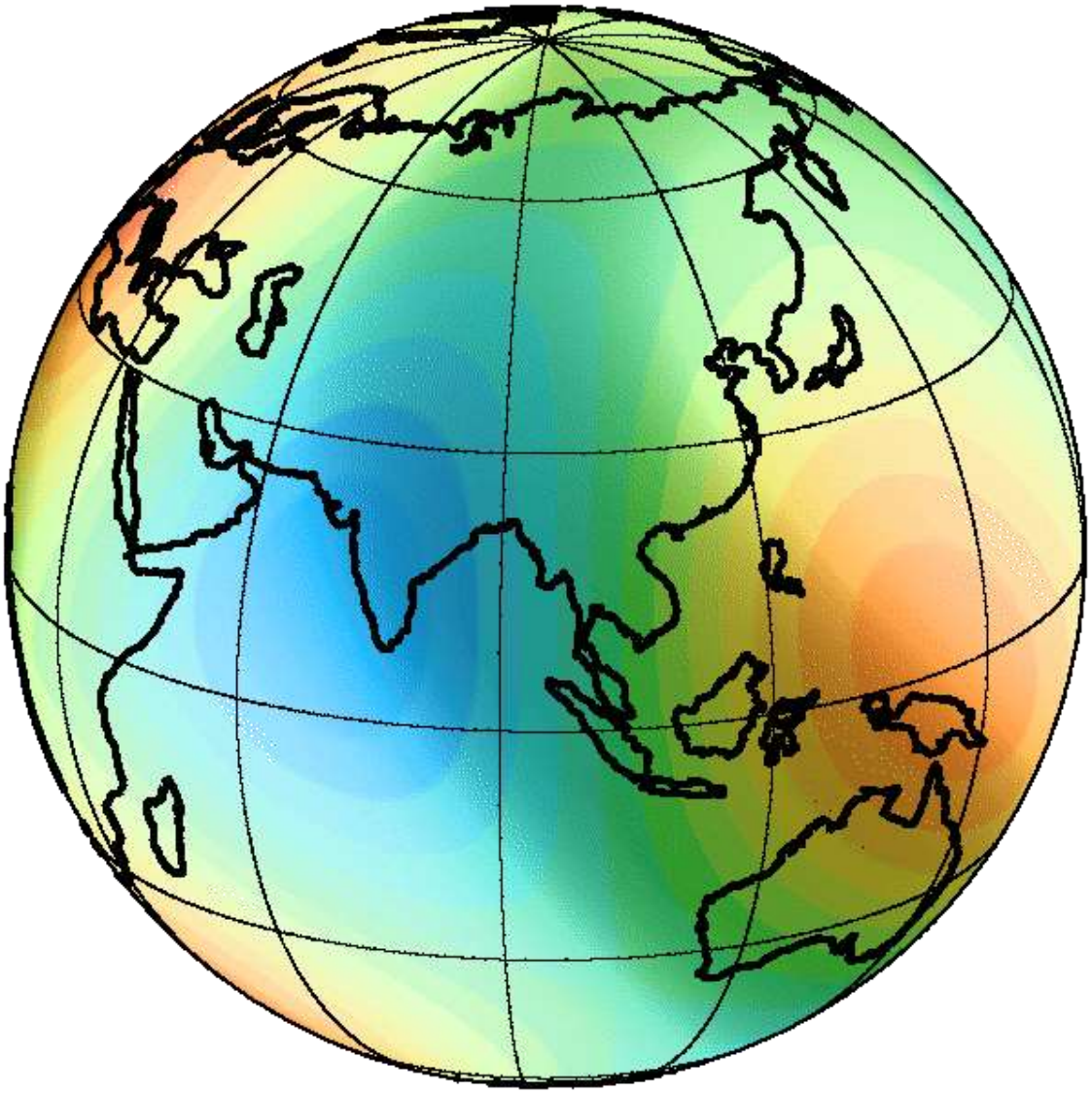
Approximation of functions on the sphere



Approximation with spherical harmonics

$$f(\mathbf{x}) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm} Y_{nm}(\mathbf{x})$$

Degree *n*	Number of coefficients
4	25
8	81
16	289
30	961
60	3721
120	14641
240	58081



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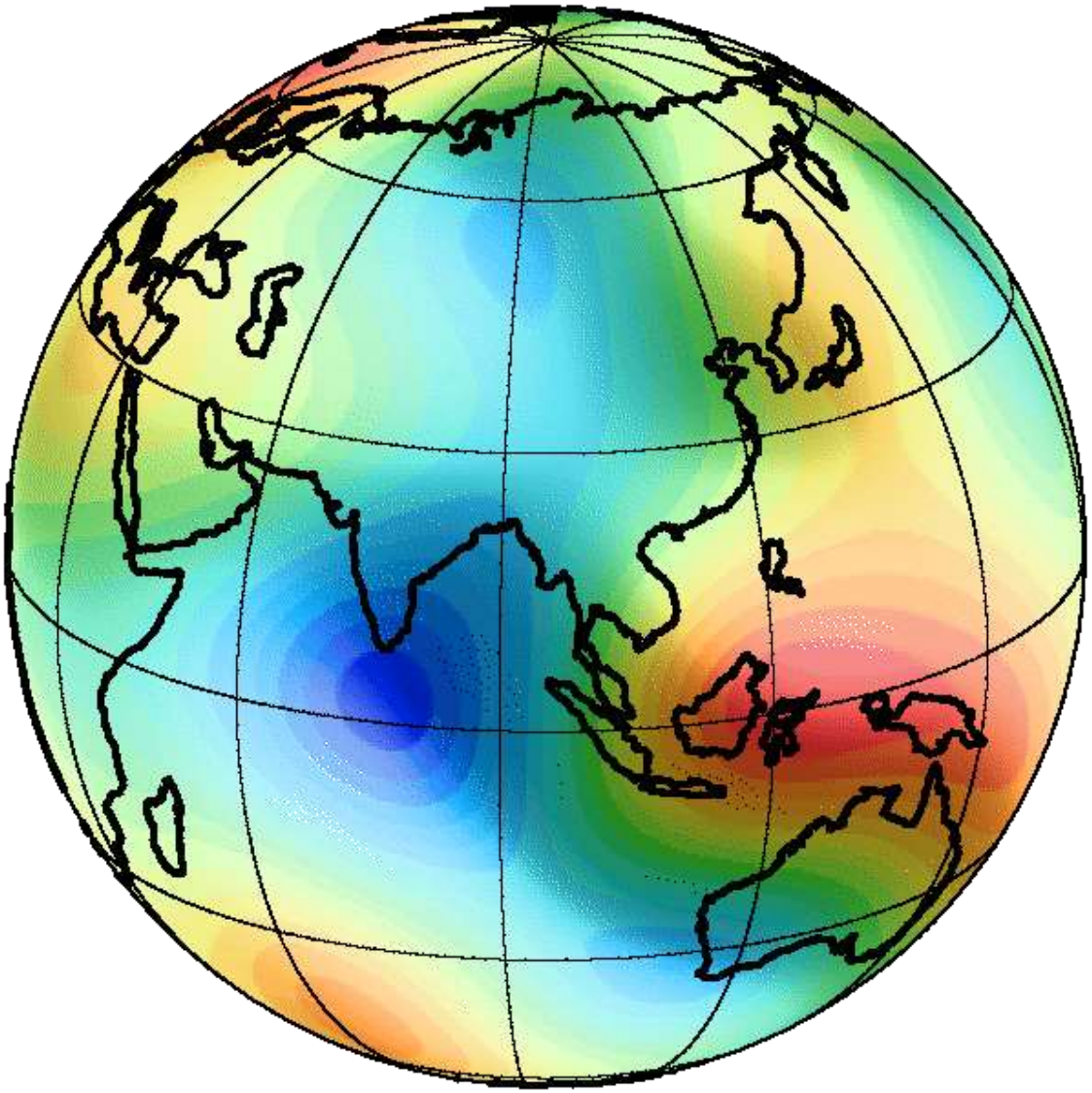

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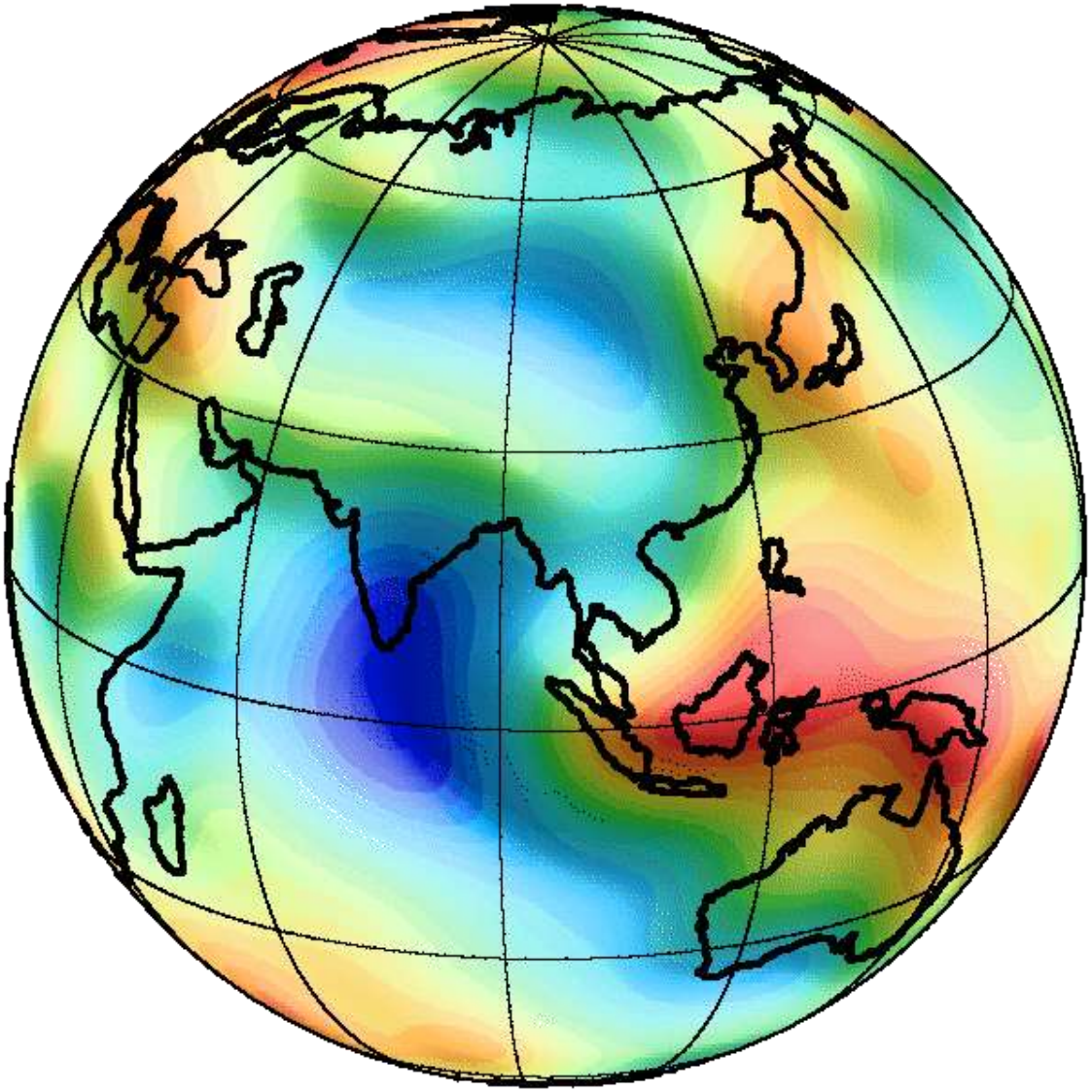
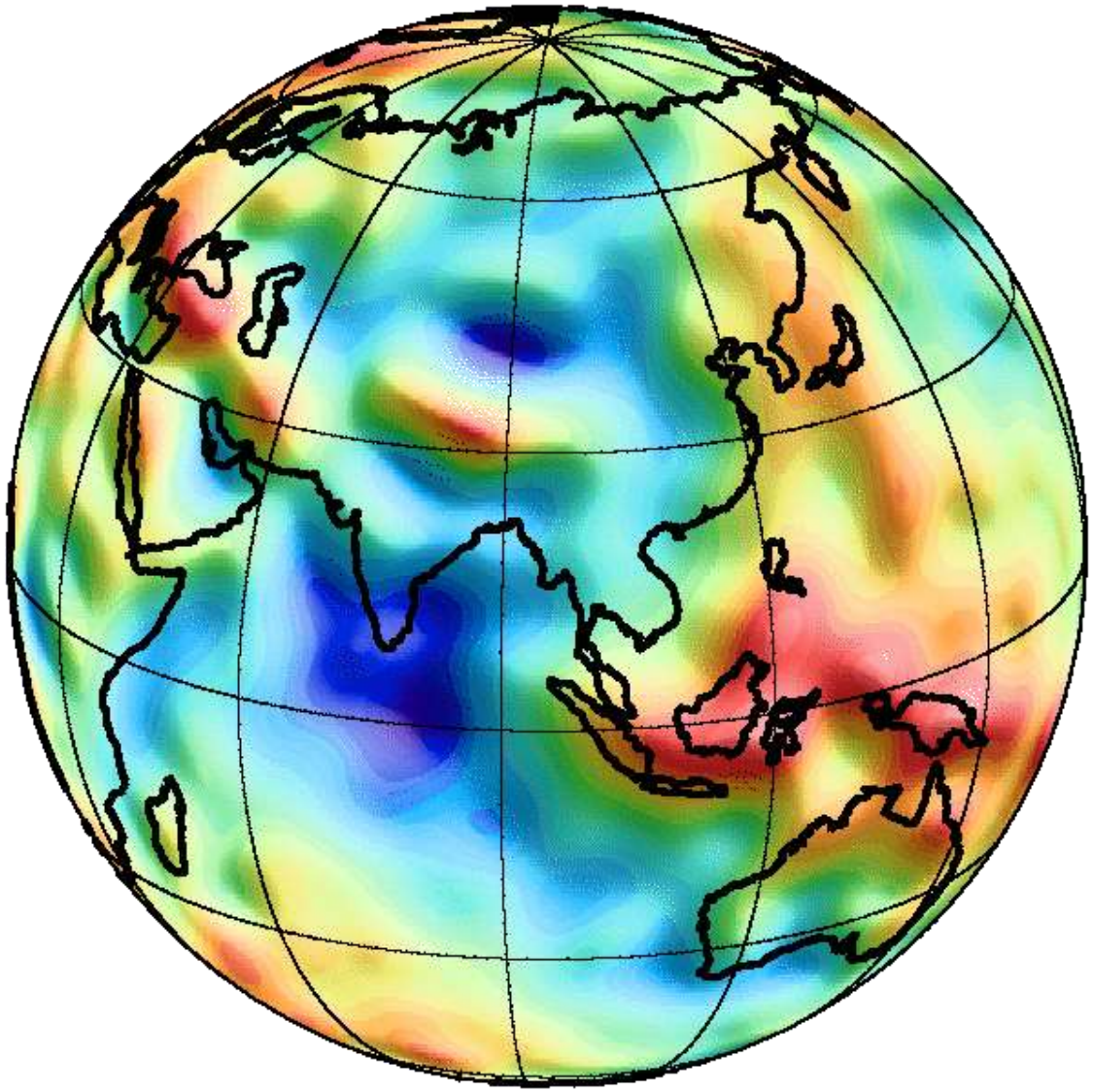
240	58081



Approximation with spherical harmonics

$$f(\mathbf{x}) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm} Y_{nm}(\mathbf{x})$$

Degree *n*	Number of coefficients
4	25
8	81
16	289
30	961
60	3721
120	14641
240	58081



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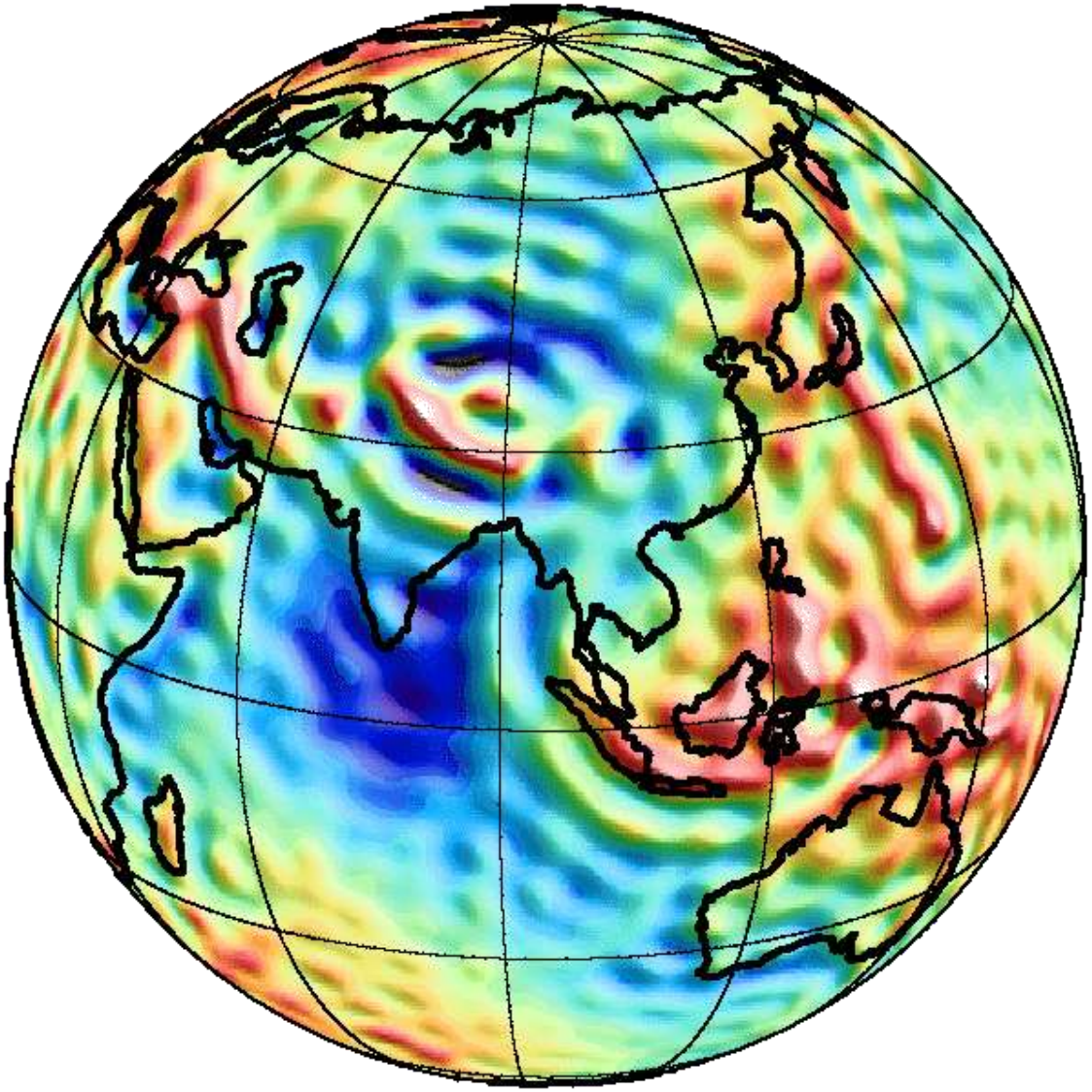

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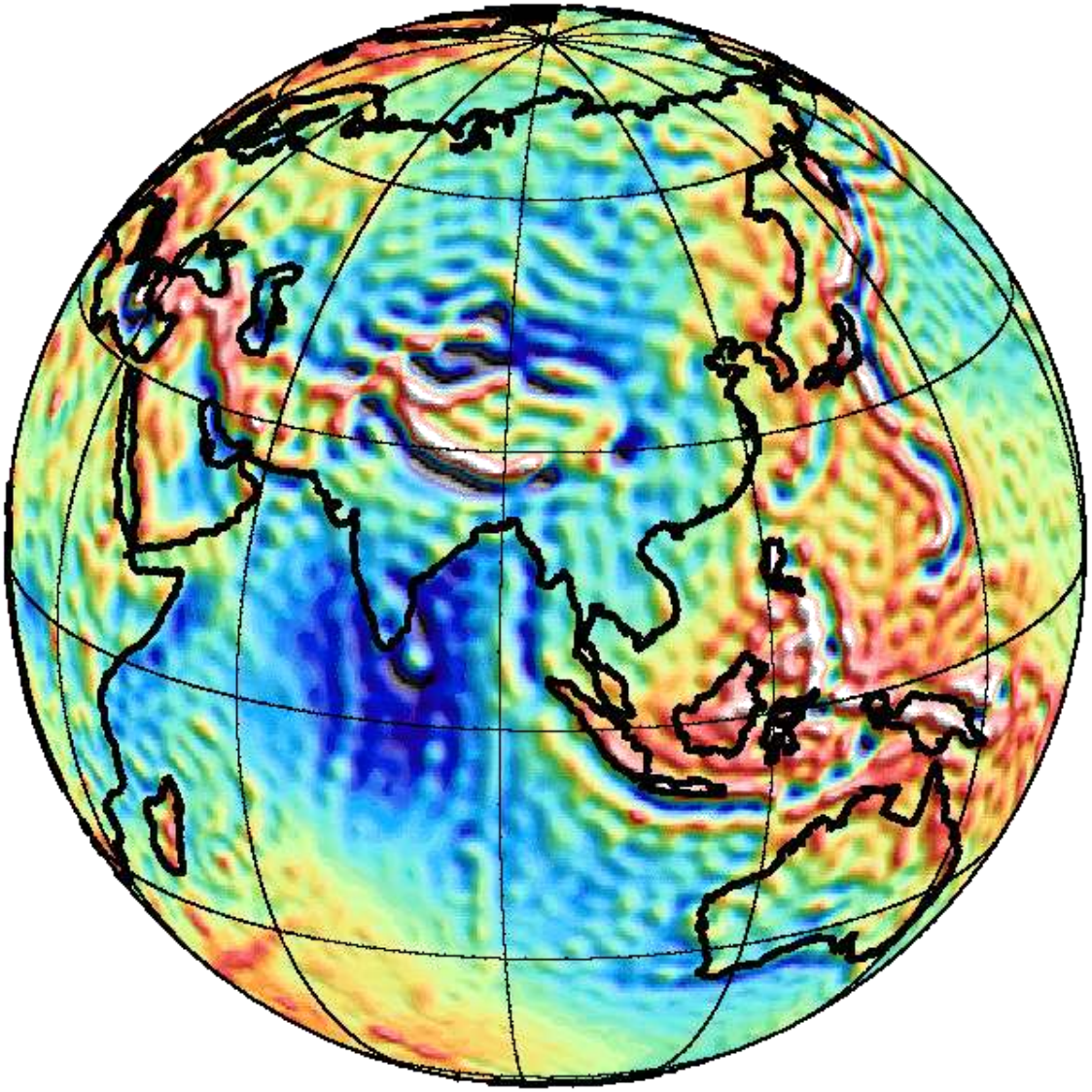

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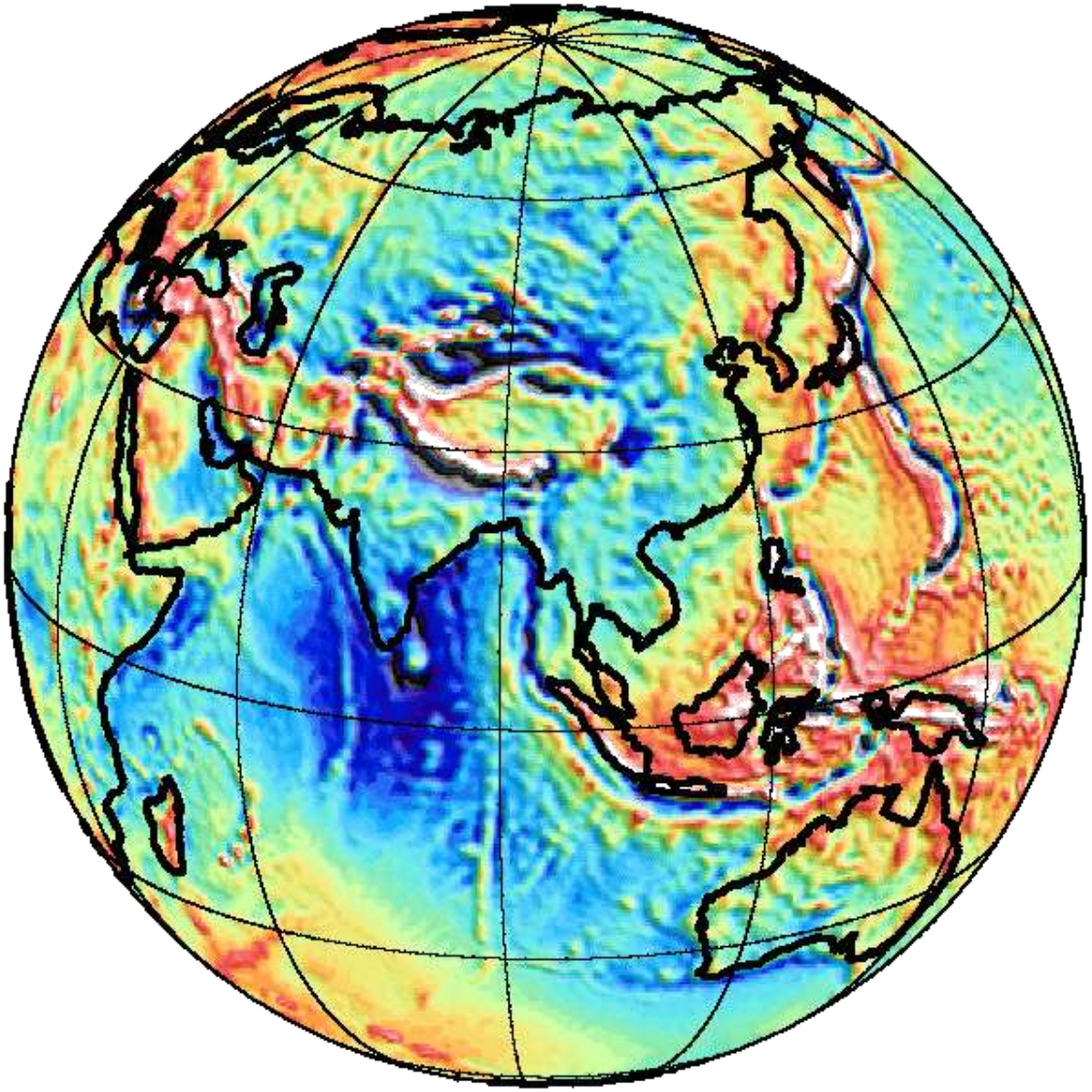

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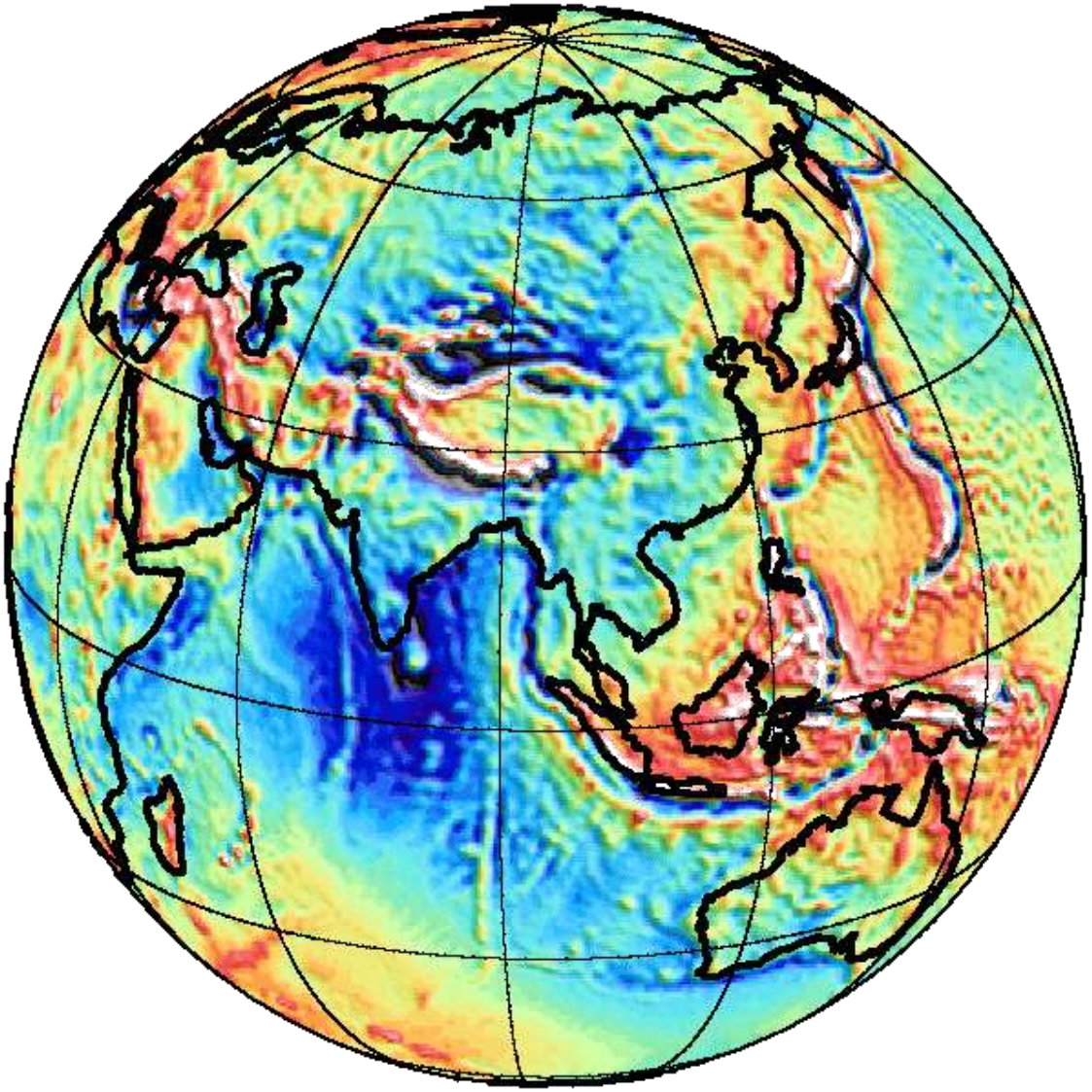


The basis functions

The basis functions

Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}Y_{nm}(\lambda,\vartheta)$$

coefficients (scale)

Basis functions

Basis functions

$$Y_{n,m}(\lambda,\vartheta)=\cos(m\lambda)\,P_{nm}(\cos\vartheta)$$

$$Y_{n,-m}(\lambda,\vartheta)=\sin(m\lambda)\,P_{nm}(\cos\vartheta)$$

depends on longitude:
Fourier series along a parallel of latitude

depends on latitude:
Legendre functions along a meridian

The basis functions

Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}Y_{nm}(\lambda,\vartheta)$$

coefficients (scale)

Basis functions

Basis functions

$$Y_{n,m}(\lambda,\vartheta)=\cos(m\lambda)\,P_{nm}(\cos\vartheta)$$

$$Y_{n,-m}(\lambda,\vartheta)=\sin(m\lambda)\,P_{nm}(\cos\vartheta)$$

Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(C_{nm}\cos(m\lambda)+S_{nm}\sin(m\lambda)\right)P_{nm}(\cos\vartheta)$$

coefficients (scale)

Computation of the basis functions

Coordinates on sphere

$$x = \cos\lambda \sin\vartheta$$
$$y = \sin\lambda \sin\vartheta$$
$$z = \cos\vartheta$$

1. $Y_{0,0} = 1$

2. 

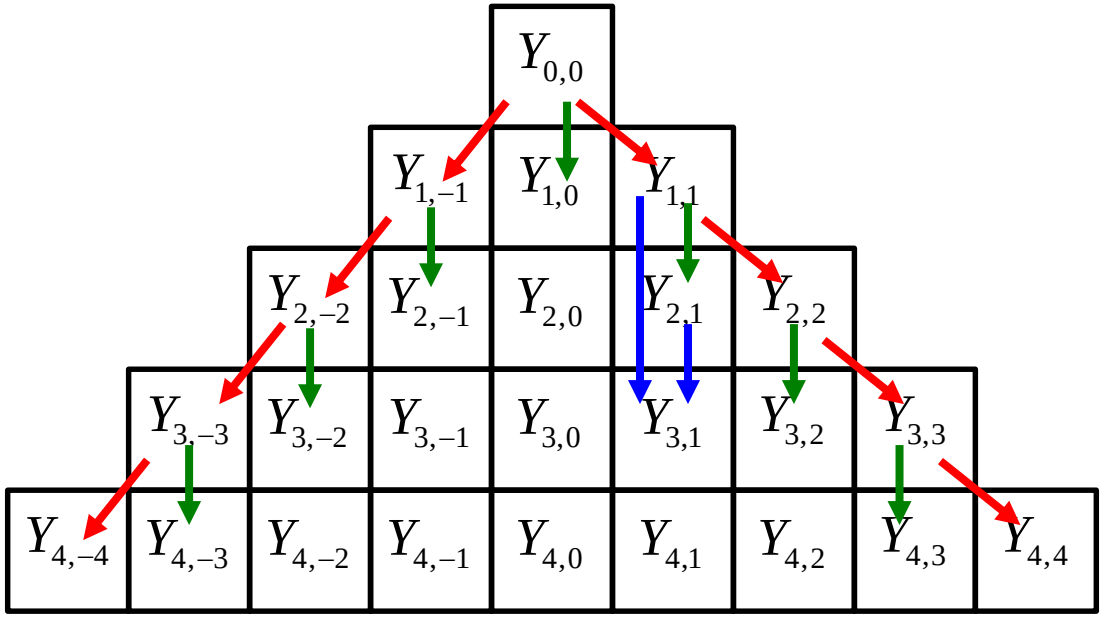
$$Y_{n,n} = a_n (x Y_{n-1,n-1} - y Y_{n-1,-n+1})$$
$$Y_{n,-n} = a_n (y Y_{n-1,n-1} + x Y_{n-1,-n+1})$$

3.  for $n = |m| + 1$

$$Y_{n,m} = b_{n,|m|}\, z\, Y_{n-1,m}$$

4.  for $n = |m| + 2, \cdots, \infty$

$$Y_{n,m} = b_{n,|m|}\, z\, Y_{n-1,m} - c_{n,|m|}\, Y_{n-2,m}$$



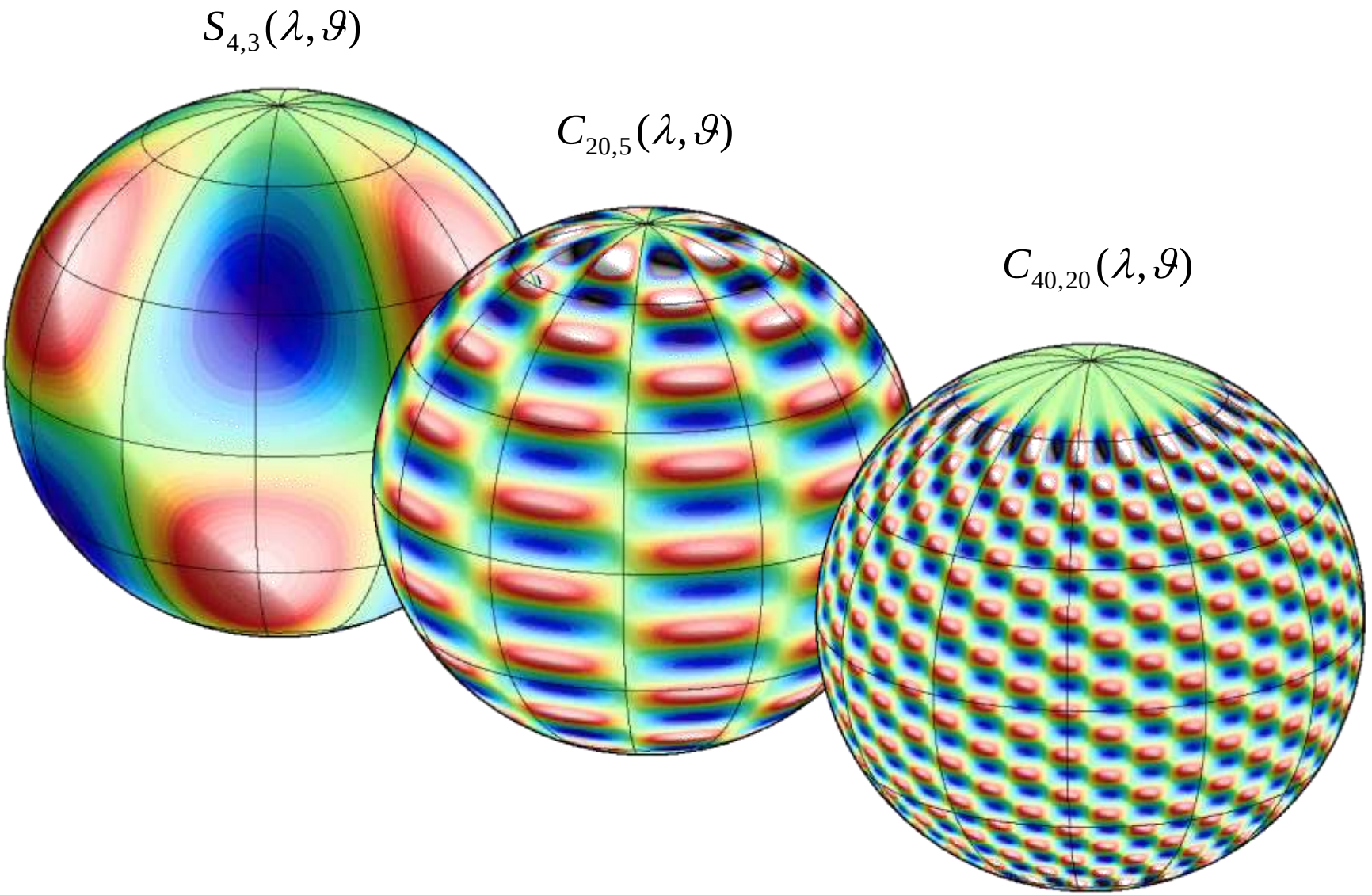
Factors (normalized)

$$a_1 = \sqrt{3} \qquad a_n = \sqrt{\frac{2n+1}{2n}}$$

$$b_{nm} = \sqrt{\frac{(2n+1)(2n-1)}{(n+m)(n-m)}}$$

$$c_{nm} = \sqrt{\frac{(2n+1)(n-m-1)(n+m-1)}{(2n-3)(n+m)(n-m)}}$$

Basis functions



Basis functions

Degree $n = 0$

Degree $n = 1$

Degree $n = 2$

Degree $n = 3$

Degree $n = 4$

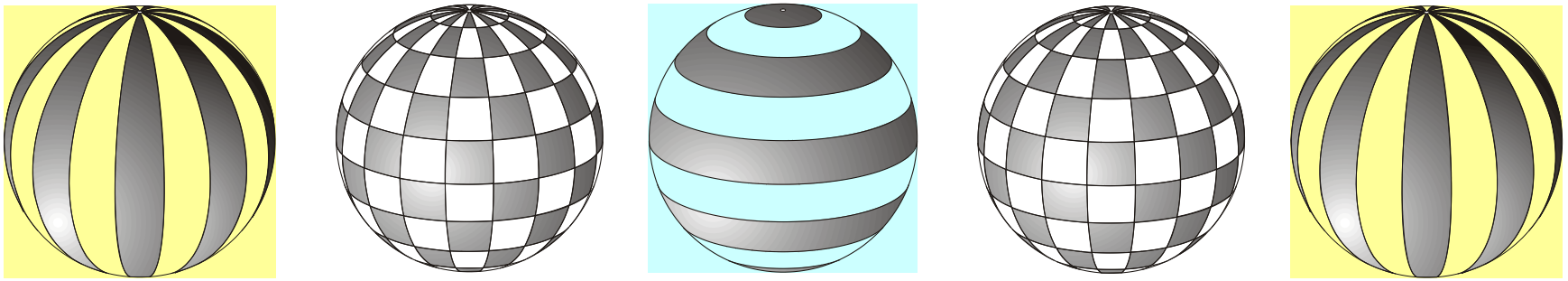
Degree $n = 5$

$$Y_{n,m}(\lambda,\vartheta) = P_n^m(\cos\vartheta)\cos m\lambda$$

$$Y_{n,-m}(\lambda,\vartheta) = P_n^m(\cos\vartheta)\sin m\lambda$$

				C_{00}						
			S_{11}	C_{10}	C_{11}					
		S_{22}	S_{21}	C_{20}	C_{21}	C_{22}				
	S_{33}	S_{32}	S_{31}	C_{30}	C_{31}	C_{32}	C_{33}			
S_{44}	S_{43}	S_{42}	S_{41}	C_{40}	C_{41}	C_{42}	C_{43}	C_{44}		
S_{55}	S_{54}	S_{53}	S_{52}	S_{51}	C_{50}	C_{51}	C_{52}	C_{53}	C_{54}	C_{55}

sektoriell | tesseral | zonal | tesseral | sektoriell



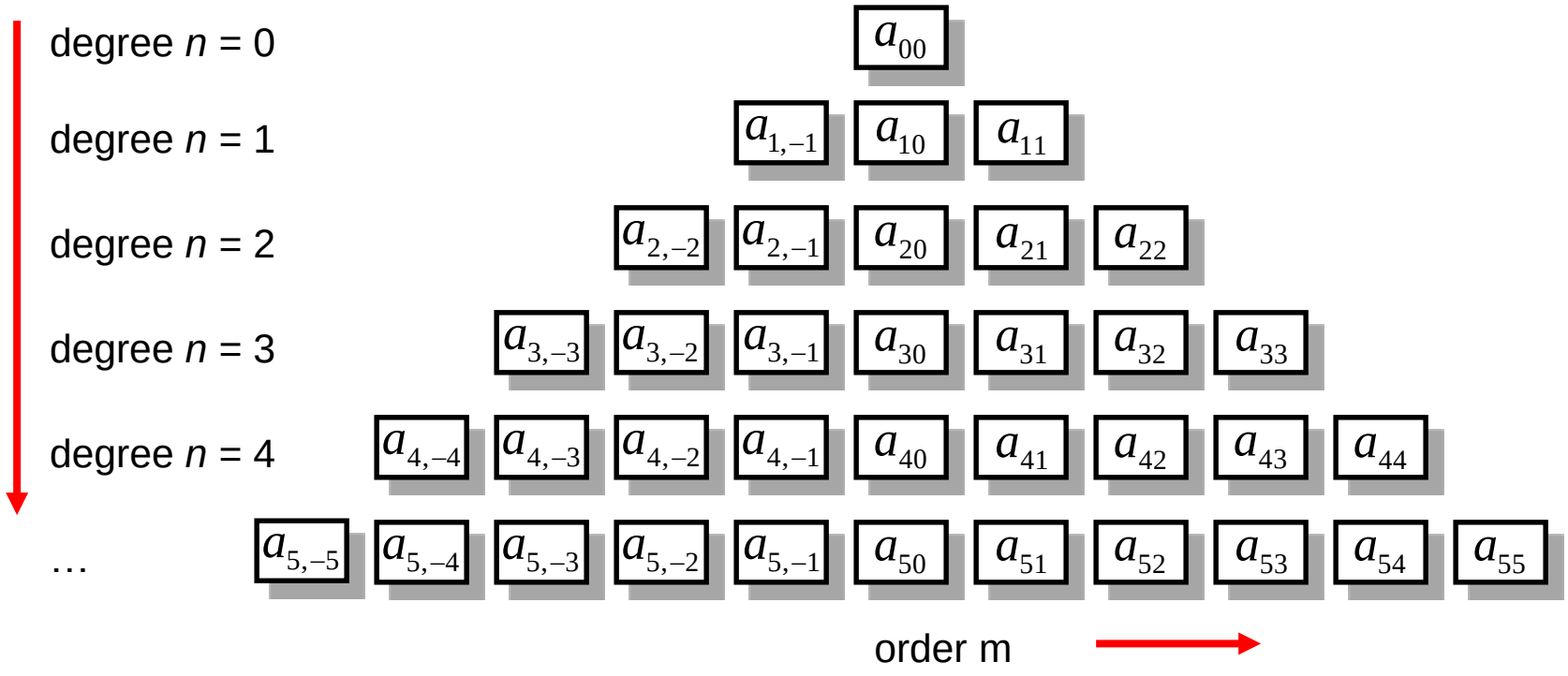
Rummel

Coefficient triangle

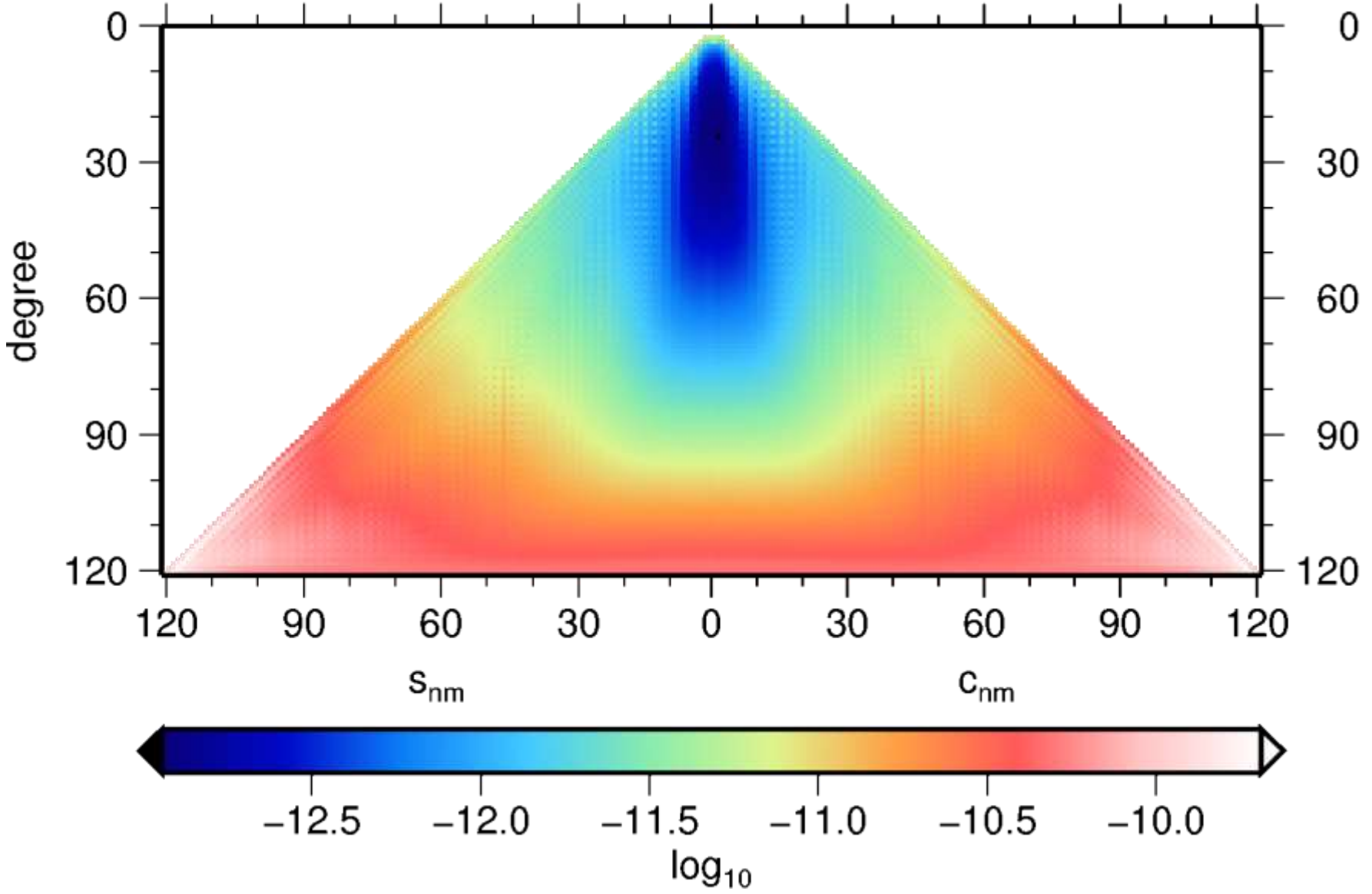
Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}Y_{nm}(\lambda,\vartheta)$$

Arrangement of coefficients in a triangle



Accuracies of coefficients



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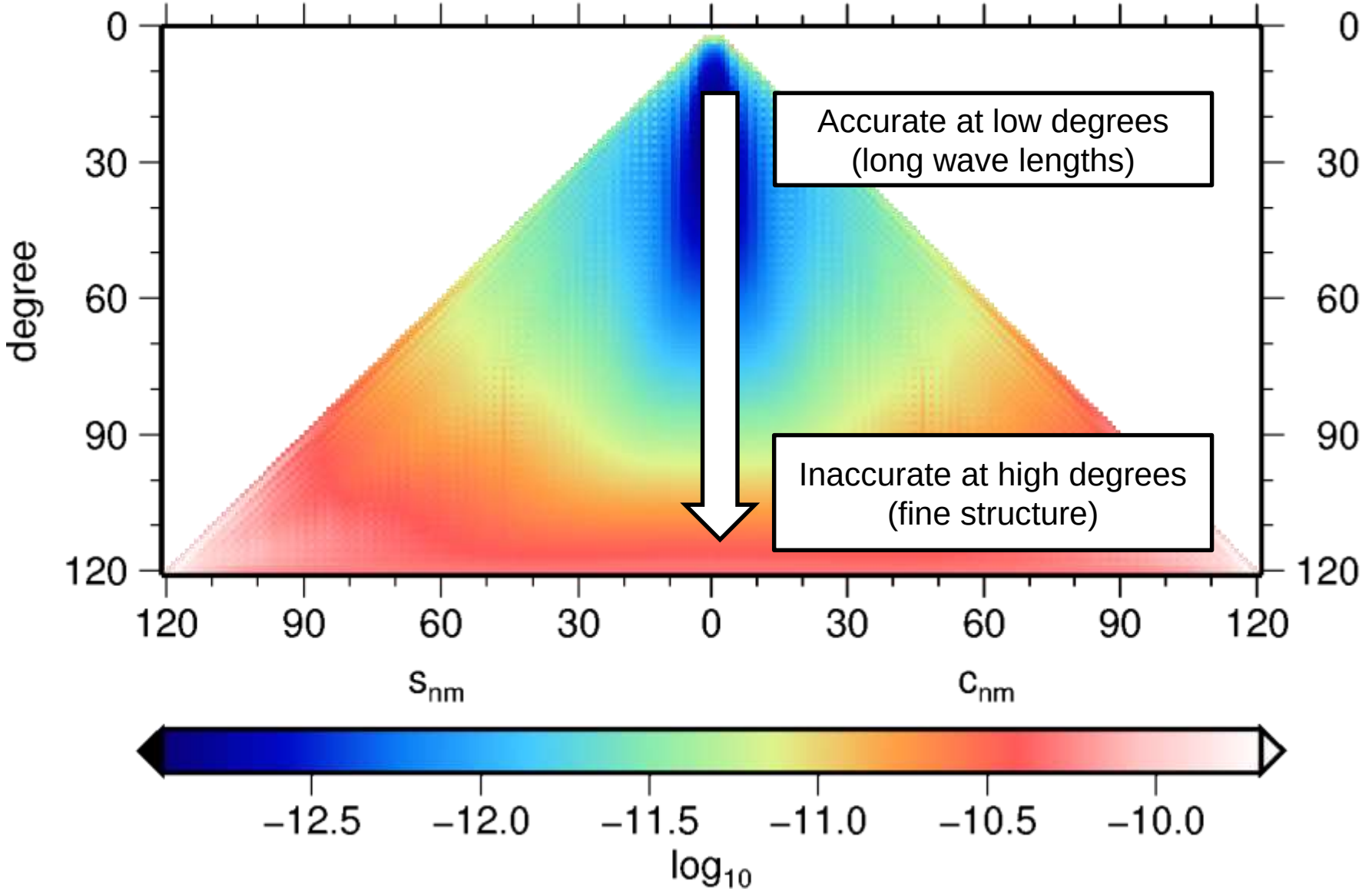
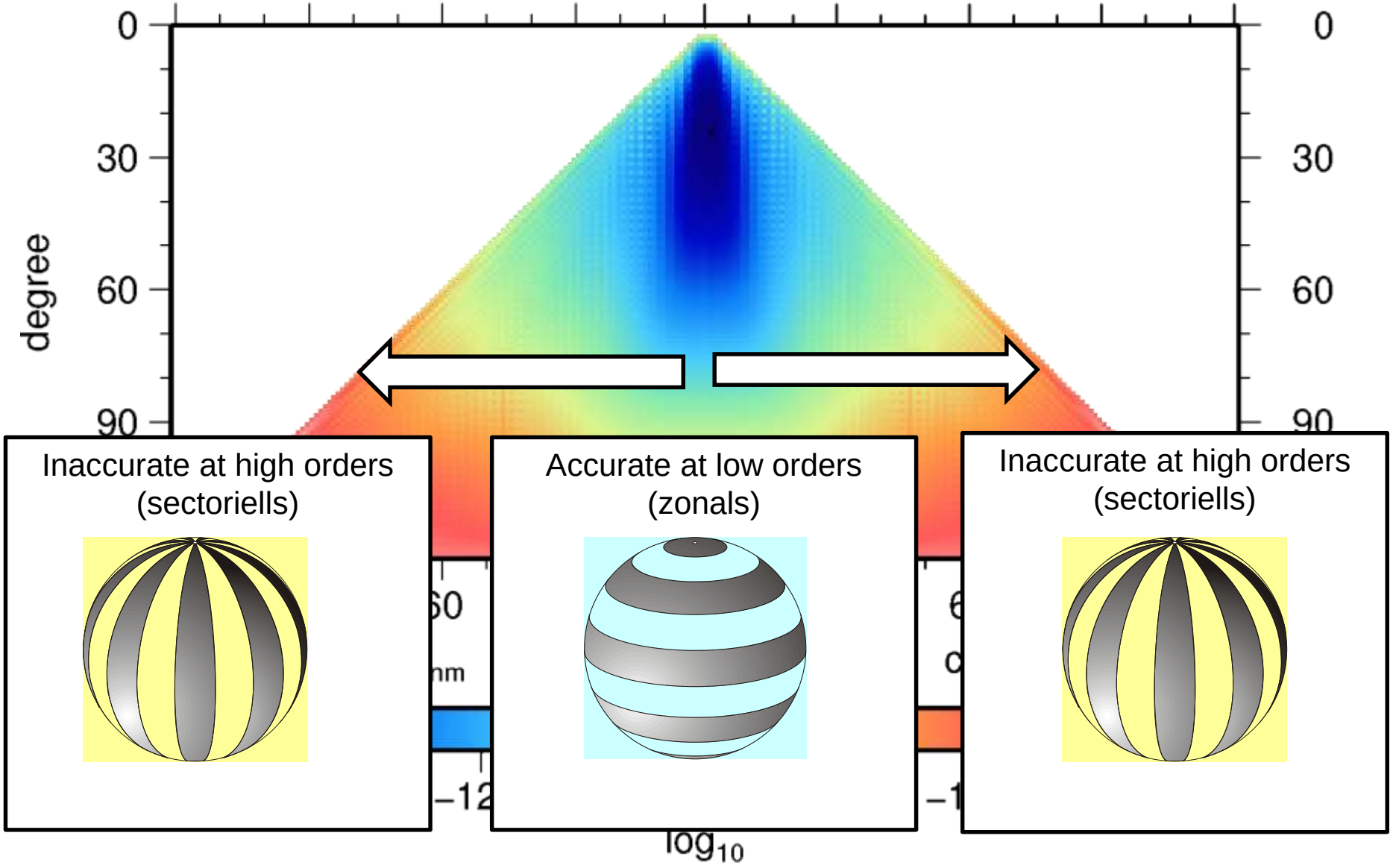


Accuracies of coefficients



Gravity field solution

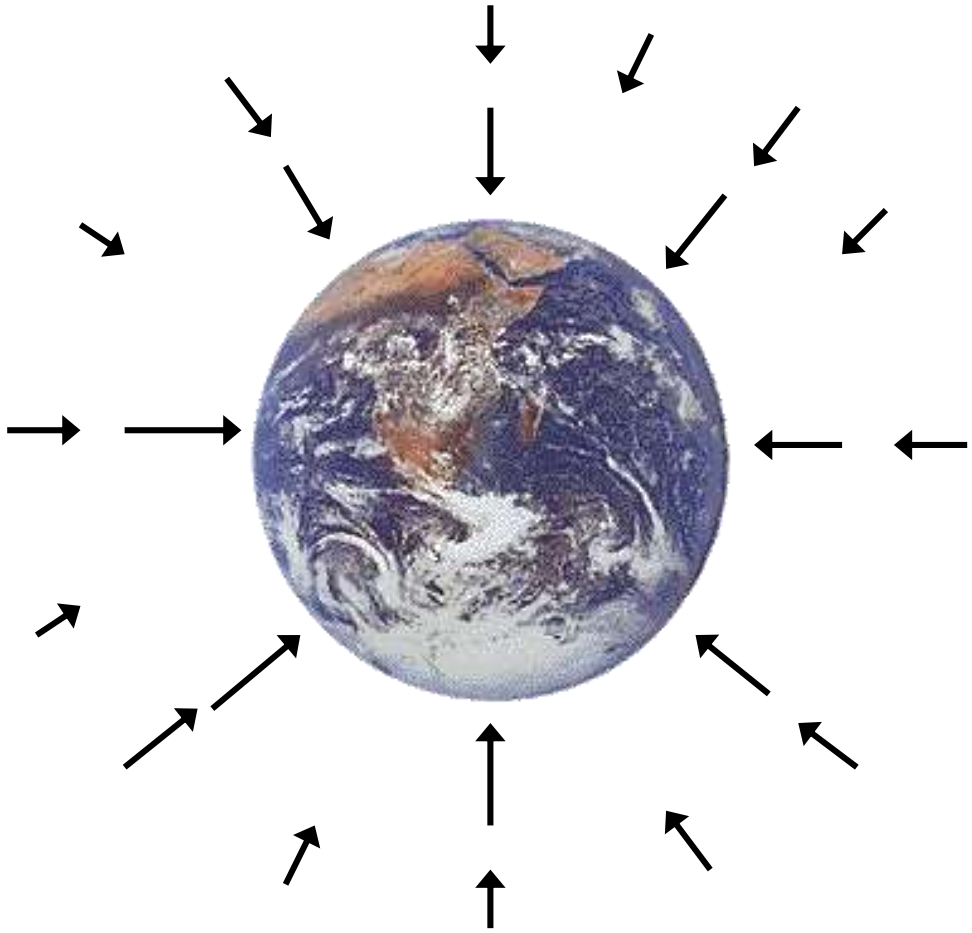
```
product_type            gravity_field
modelname               ITG-Grace03
comment                 static field from 2002-08 to 2007-04 of GRACE data
earth_gravity_constant 3.986004415e+14
radius                  6378136.6
max_degree              180
key n m        C                   S                   sigma C             sigma S
end_of_head ===================================================================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 1 1  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000000000000e+00
gfc 2 0 -4.841692718699e-04  0.000000000000e+00 6.469883774458e-13 0.000000000000e+00
gfc 2 1 -2.654790999243e-10  1.475393314283e-09 6.108979511966e-13 6.355307212507e-13
gfc 2 2  2.439383367978e-06 -1.400273635220e-06 6.254221806143e-13 6.423410956098e-13
gfc 3 0  9.571610348416e-07  0.000000000000e+00 4.908157850872e-13 0.000000000000e+00
gfc 3 1  2.030461736678e-06  2.482003394707e-07 4.904543816334e-13 5.118675157415e-13
gfc 3 2  9.047877724984e-07 -6.190053685183e-07 5.459595906001e-13 5.482674767117e-13
gfc 3 3  7.213217237276e-07  1.414349090196e-06 5.163836126113e-13 5.163483433061e-13
gfc 4 0  5.399657665980e-07  0.000000000000e+00 3.758481731782e-13 0.000000000000e+00
gfc 4 1 -5.361573220519e-07 -4.735672404588e-07 3.874699557956e-13 3.973550735355e-13
gfc 4 2  3.505015650151e-07  6.624798955603e-07 4.501829959710e-13 4.398486563277e-13
gfc 4 3  9.908565738322e-07 -2.009566568843e-07 4.776067657084e-13 4.766590461153e-13
gfc 4 4 -1.885196275153e-07  3.088038091544e-07 4.556511148108e-13 4.565287154679e-13
gfc 5 0  6.867029195170e-08  0.000000000000e+00 2.444626602968e-13 0.000000000000e+00
gfc 5 1 -6.292117216708e-08 -9.436975416042e-08 2.510438328896e-13 2.640391465060e-13
...
```

Gravity field

Gravity field

Gravity field:

$$\mathbf{g}(\mathbf{r}) = \begin{pmatrix} g_x(\mathbf{r}) \\ g_y(\mathbf{r}) \\ g_z(\mathbf{r}) \end{pmatrix} \qquad \left[\frac{m}{s^2}\right]$$



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$$\mathbf{g}(\mathbf{r}) = \begin{pmatrix} g_x(\mathbf{r}) \\ g_y(\mathbf{r}) \\ g_z(\mathbf{r}) \end{pmatrix} \qquad \left[\frac{m}{s^2}\right]$$

Conservative vector field ⇔
Potential function exists:

$$\mathbf{g}(\mathbf{r}) = \nabla V(\mathbf{r}) = \begin{pmatrix} \partial V/\partial x \\ \partial V/\partial y \\ \partial V/\partial z \end{pmatrix} \qquad V(\mathbf{r}) \left[\frac{m^2}{s^2}\right]$$

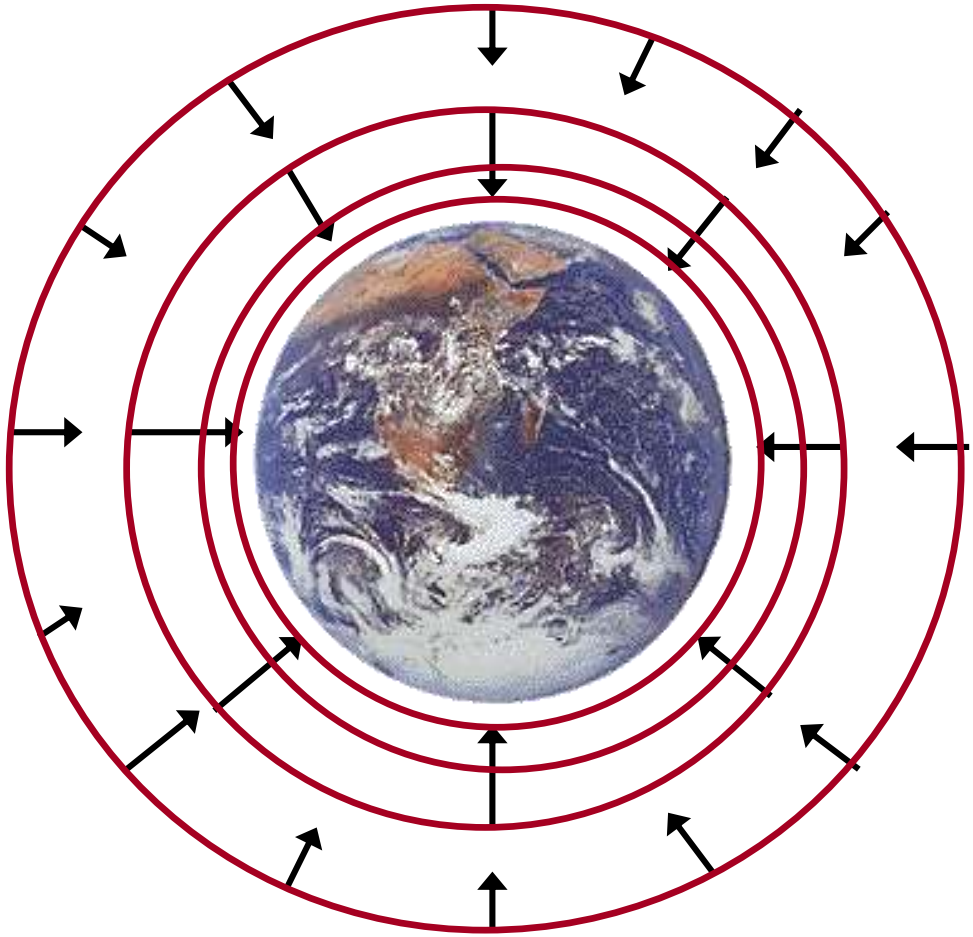
Source free in outer space

$$\operatorname{div} \mathbf{g} = \nabla \cdot \mathbf{g} = 0$$

Laplace equation (harmonic func.)

$$\nabla \cdot \nabla V = \Delta V = 0$$

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$



Harmonics continuation

Potential in outer space

$$V(\lambda,\vartheta,r)=\frac{R\left(R^2-r^2\right)}{4\pi}\iint_{\Omega}\frac{1}{l^3}V(\lambda,\vartheta)d\Omega$$

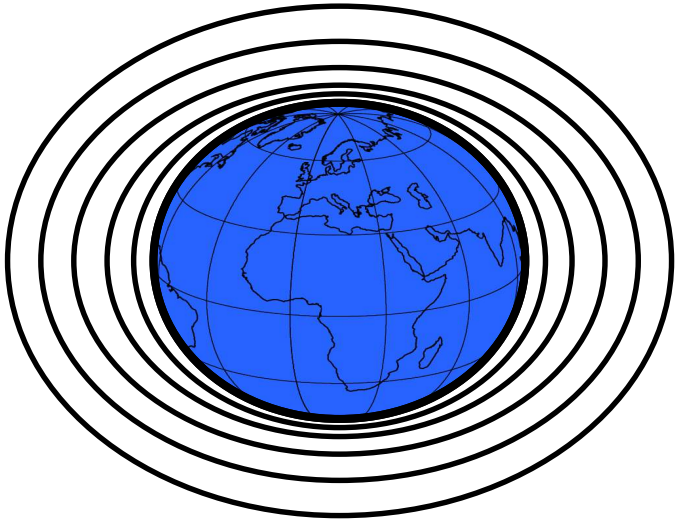
with $l=\sqrt{R^2+r^2-2Rr\cos\psi}$

Harmonic continuation

$$\Delta V=\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

Potential at Earth‘s surface

$$V(\lambda,\vartheta)\big|_{\Omega}$$



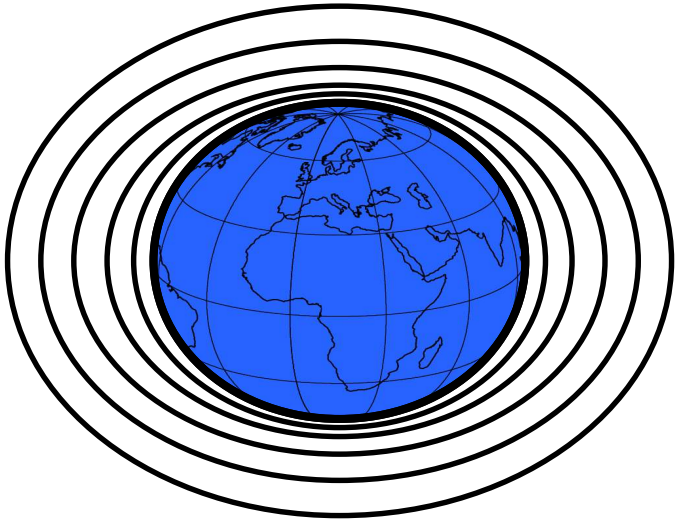
Harmonics continuation

Potential in outer space
(solid spherical harmonics)

$$V(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

Harmonic continuation

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$



Potential at Earth‘s surface
(in spherical harmonics)

$$V(\lambda,\vartheta) = \frac{GM}{R}\sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

Orders of magnitude

Gravity

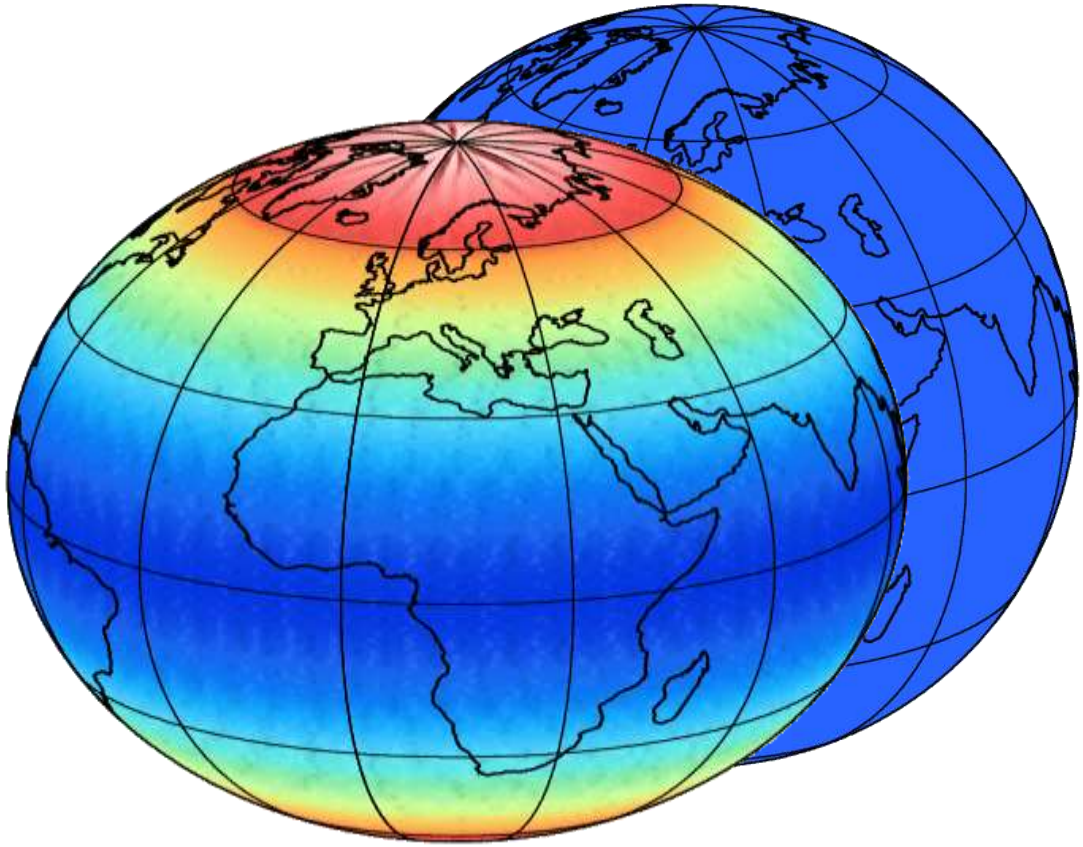
Gravity at Earth surface

$$g = 9{,}81 \frac{m}{s^2}$$

Gravity

Gravity at Earth surface

$$g = 9{,}78 \frac{m}{s^2} \ldots 9{,}83 \frac{m}{s^2}$$

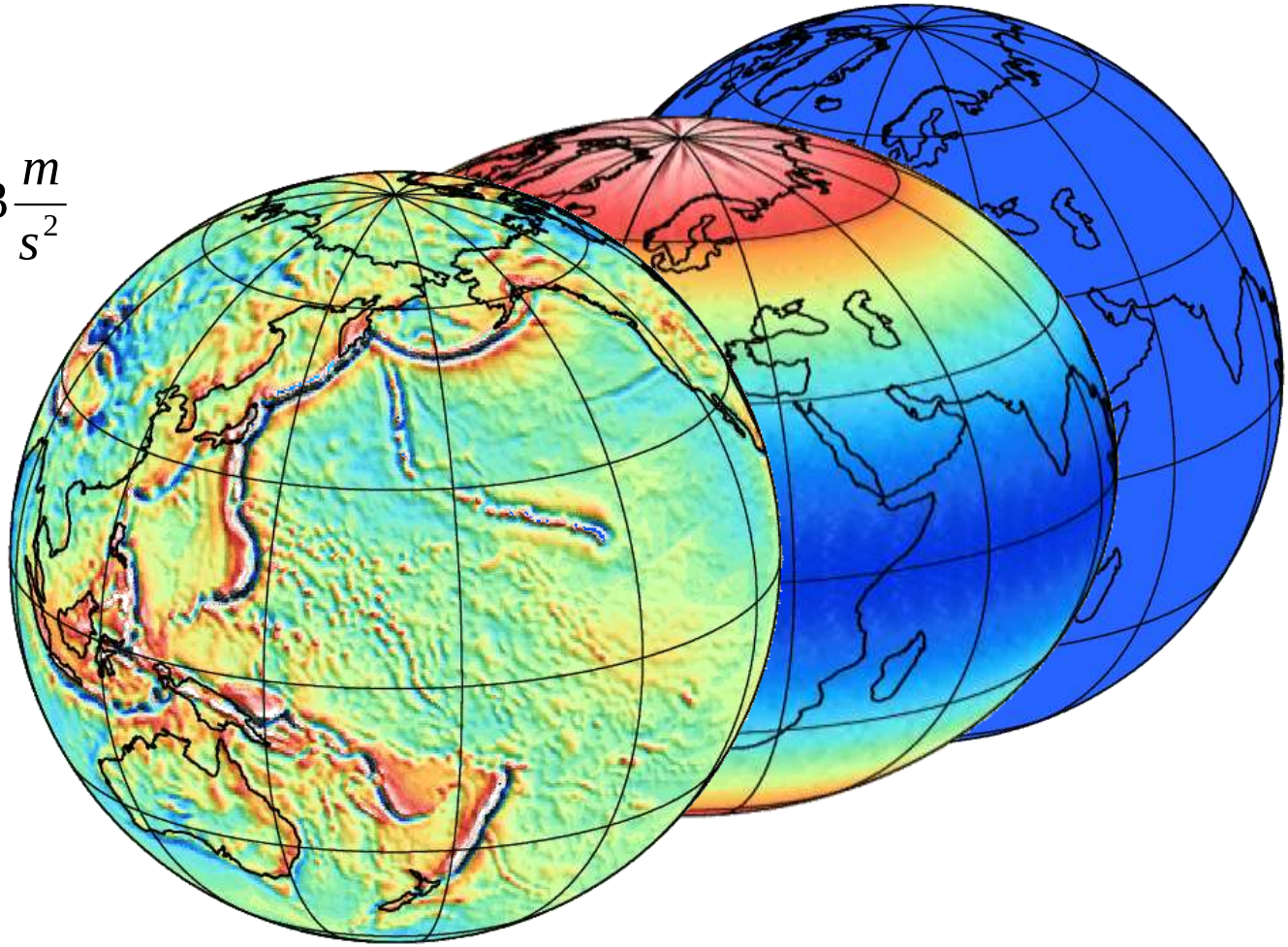


Gravity

Gravity at Earth surface

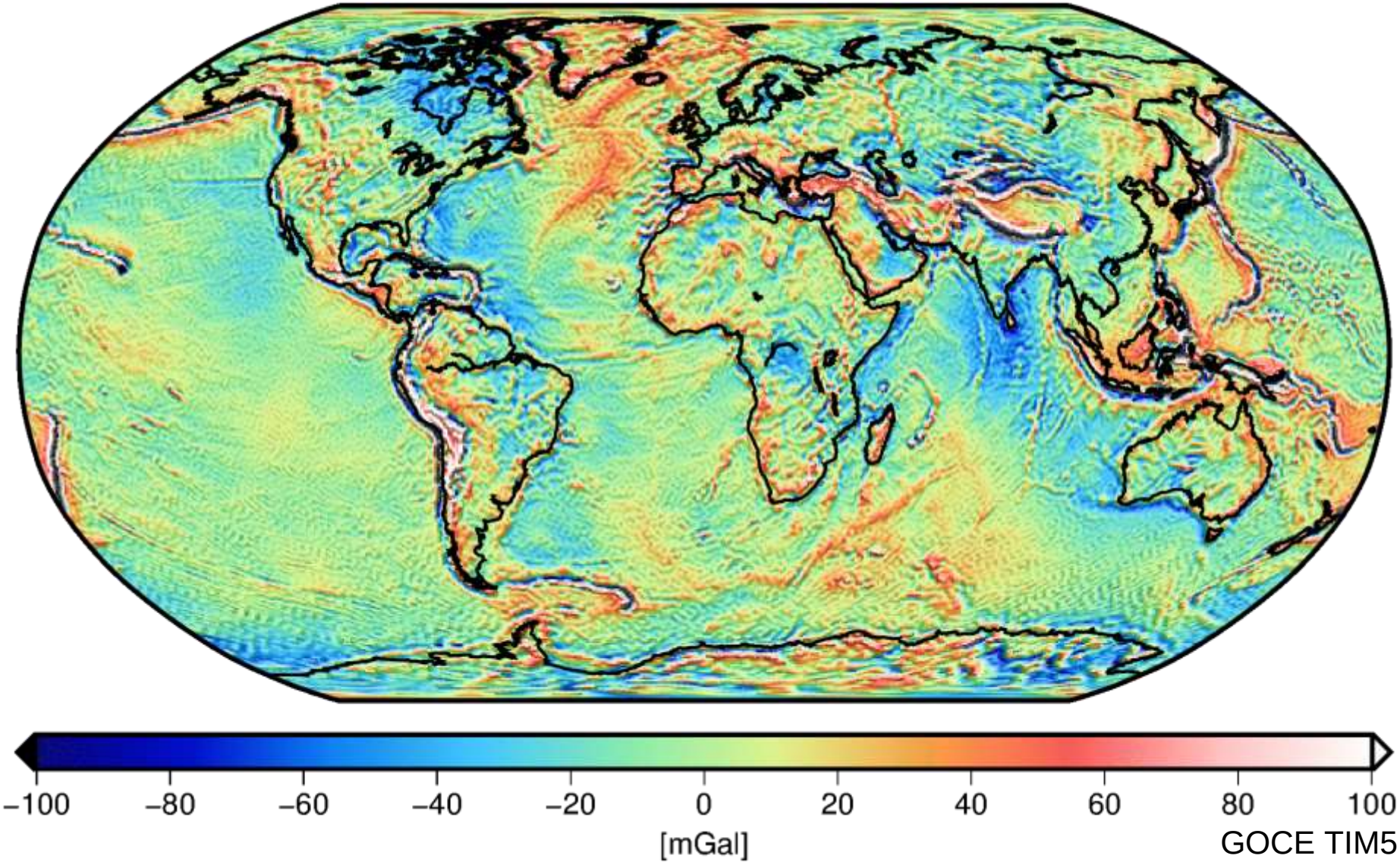
$$g = 9{,}78 \frac{m}{s^2} \ldots 9{,}83 \frac{m}{s^2}$$

$$\pm 0{,}0004 \frac{m}{s^2}$$

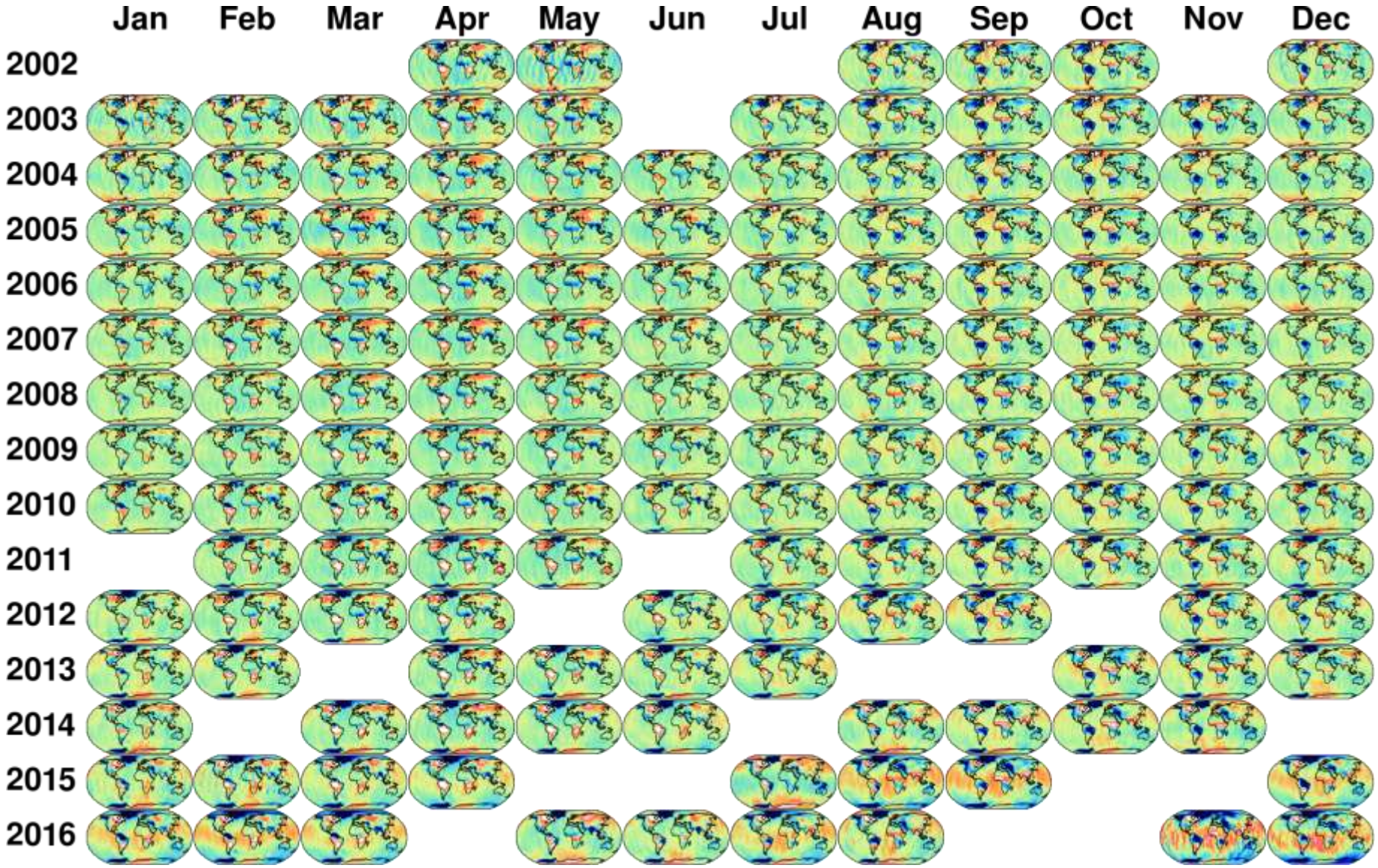


$$\left[1\text{mGal} = 0{,}00001 \frac{m}{s^2} \right]$$

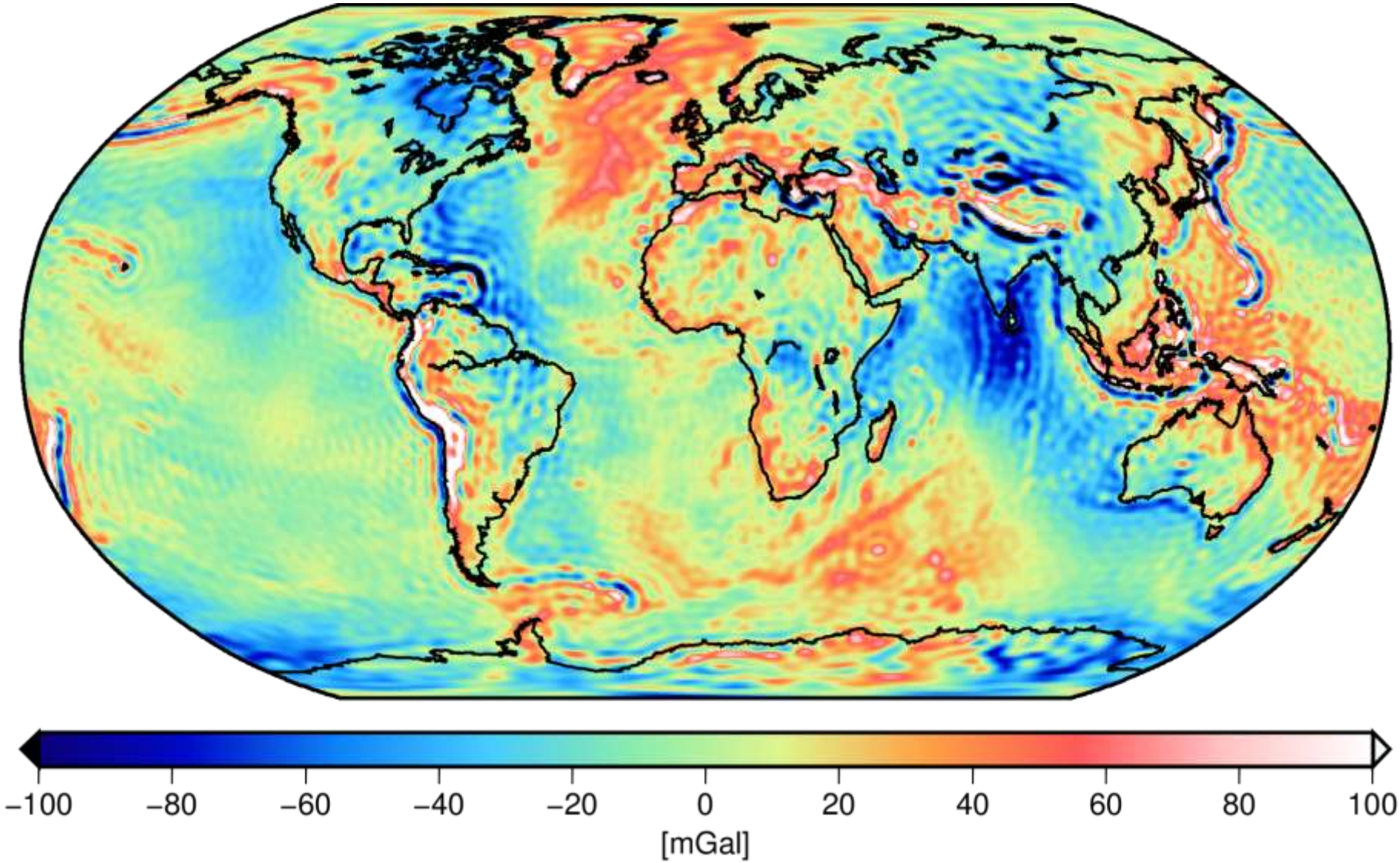
Gravity anomalies



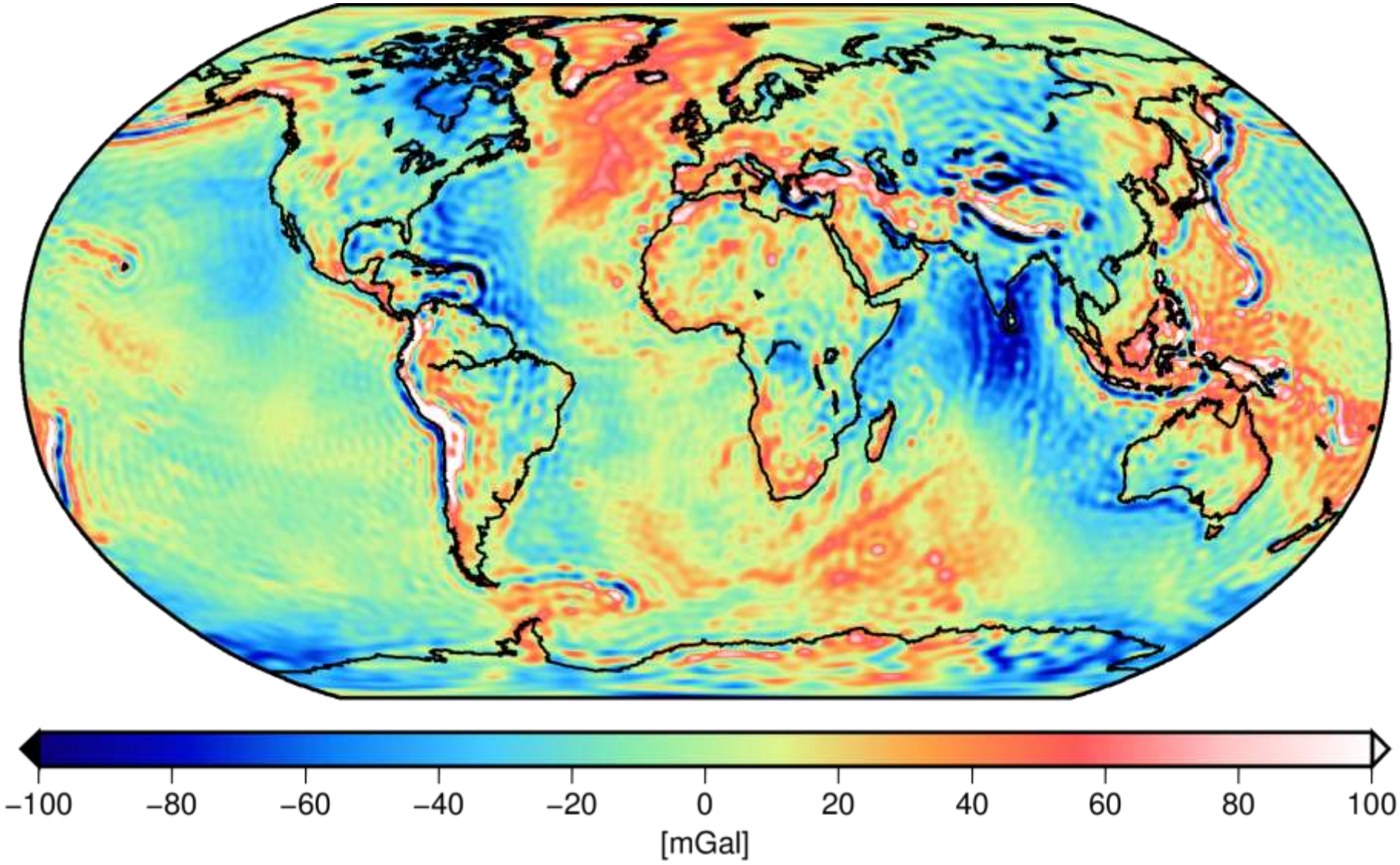
GRACE Monthly solutions



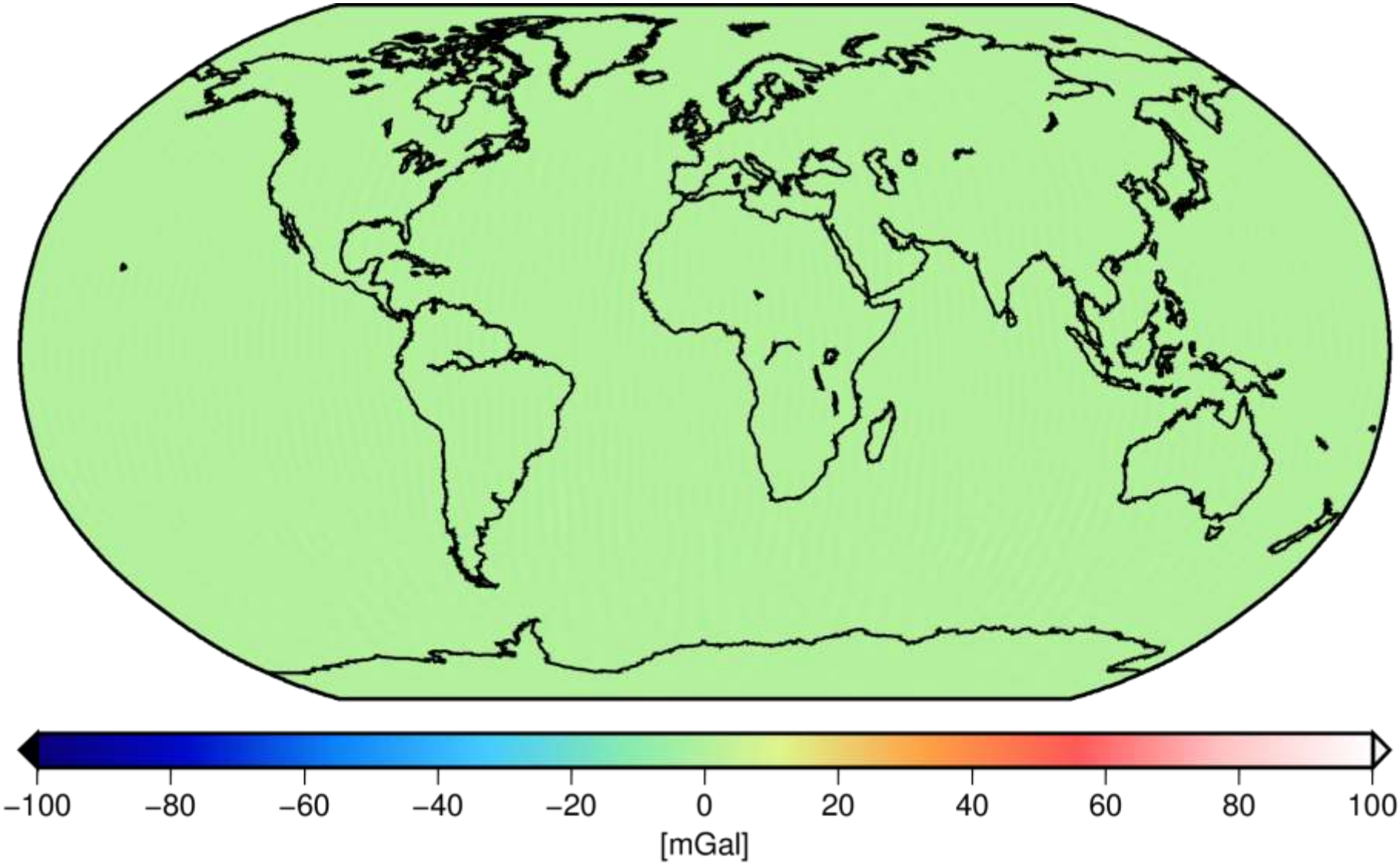
GRACE gravity field March 2008



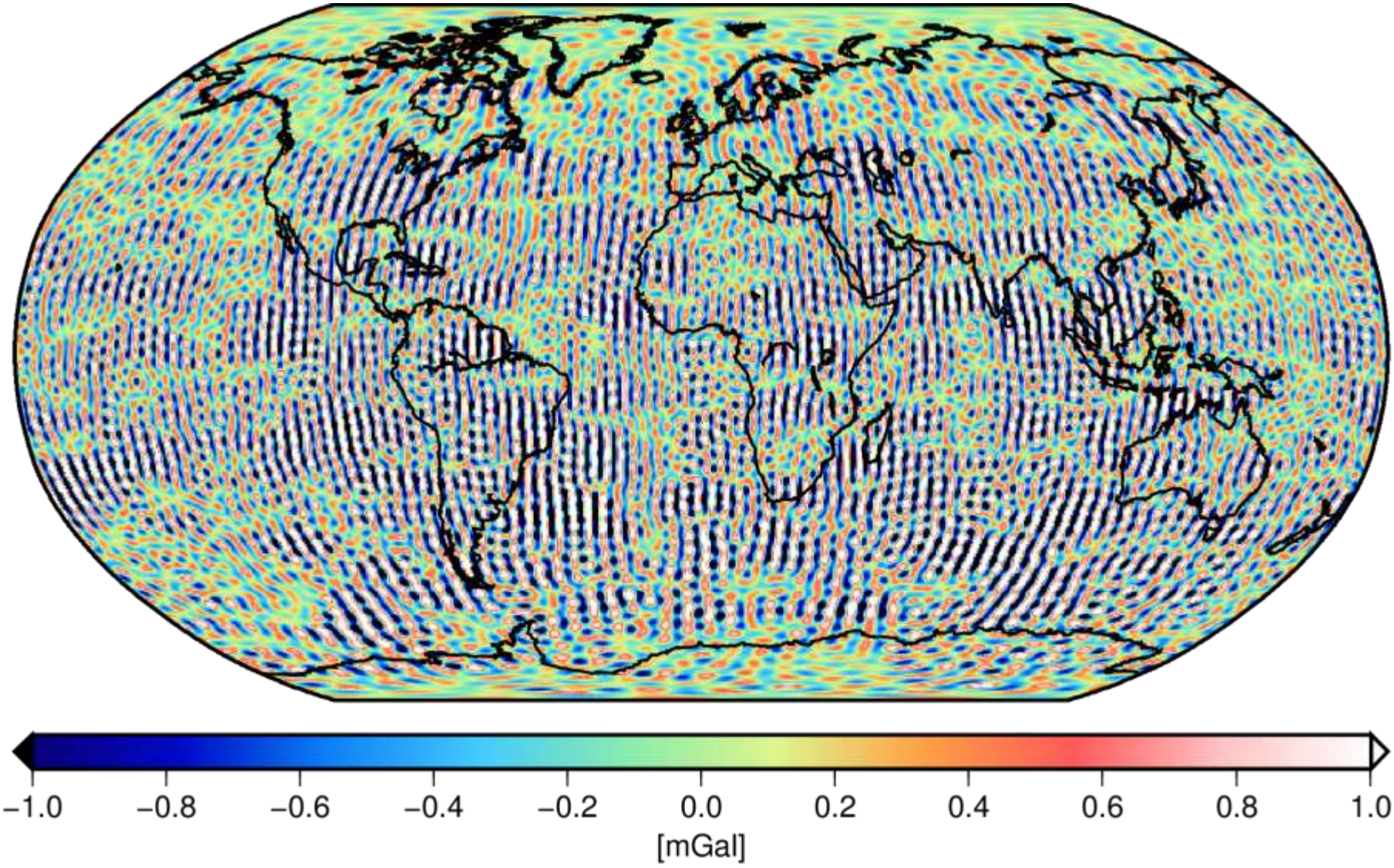
GRACE gravity field September 2008



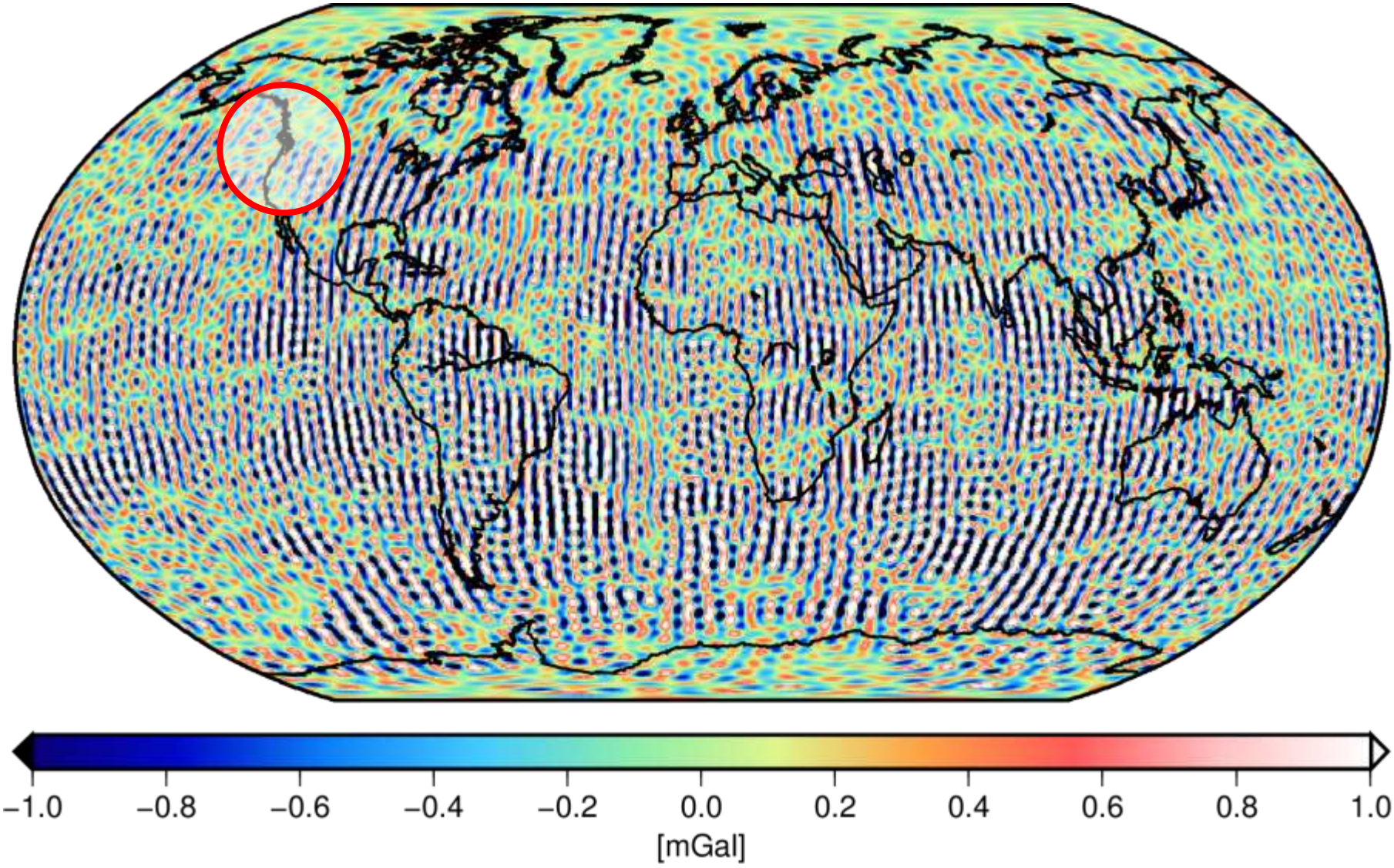
September – March 2008



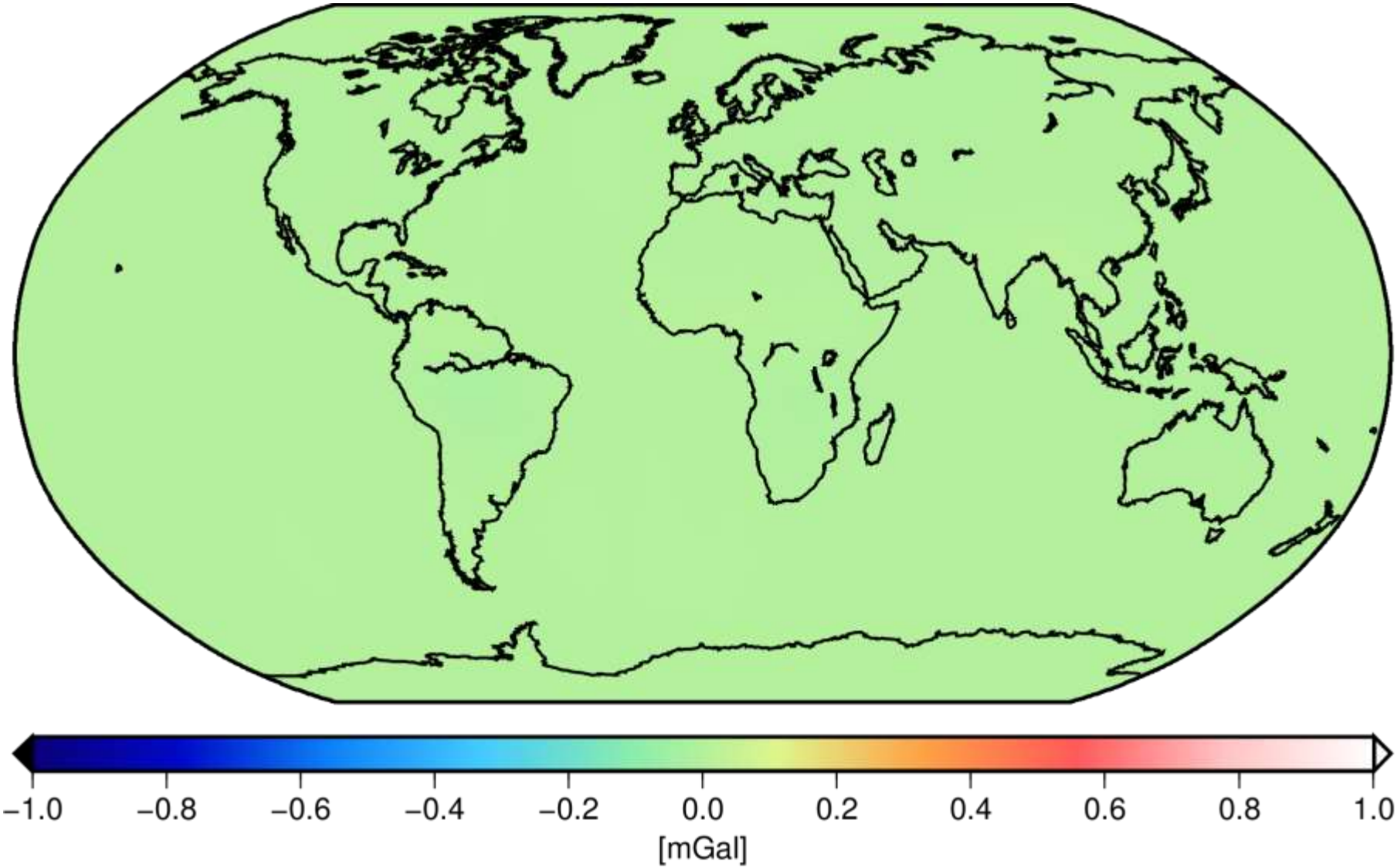
September – March 2008



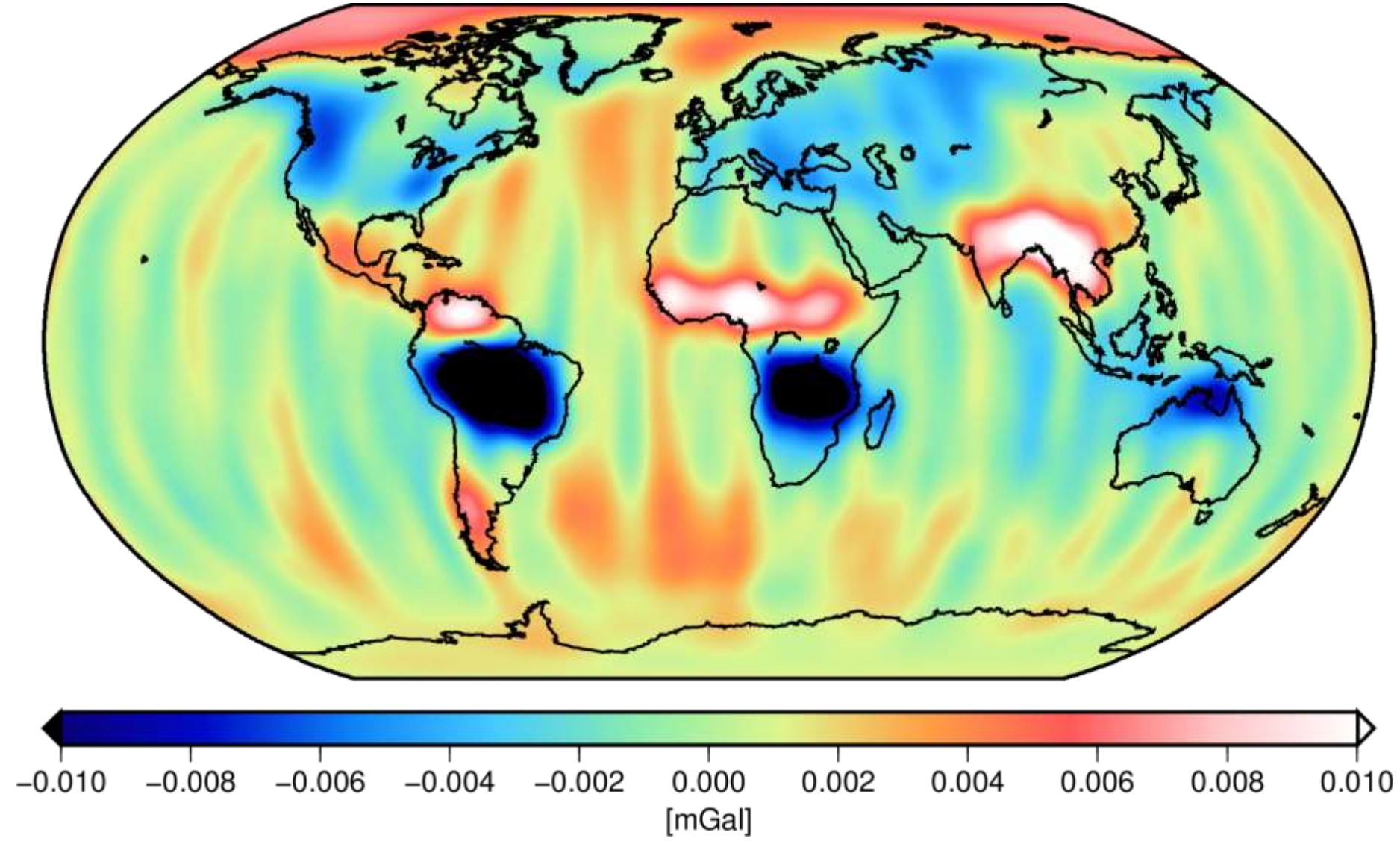
September – March 2008



September – March 2008



September – March 2008

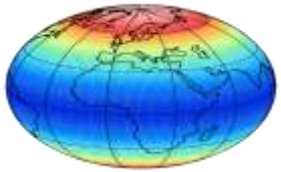


Gravity field

Sphere

$$g = \frac{GM}{R^2} \approx 10 \frac{m}{s^2} \qquad C_{00} = 1$$

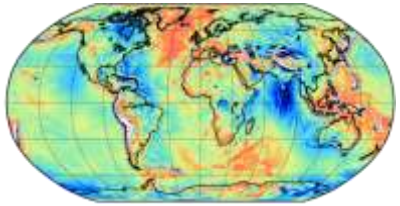
Flattening



$$g = \pm 0.03 \frac{m}{s^2} \qquad C_{20} = -0.0004842$$

Normal field (Geodetic Reference System, GRS80)

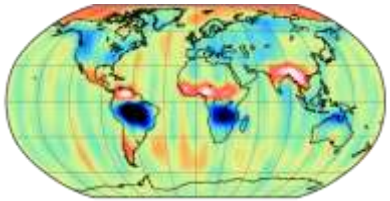
Static part



$$\delta g \approx 0.001 \frac{m}{s^2} = 100 \text{mGal}$$

Spherical approximations + ellipsoidal corrections

Time variable part



$$\Delta \delta g \approx 0.0000001 \frac{m}{s^2} = 0.01 \text{ mGal}$$

Spherical approximations

Functionals of the gravity field

Gravity field functionals

Disturbing potential

$$T = V - U$$

Normal potential

$$T(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

gravity

$$g = \|\nabla V\|$$

Geoid heights

$$V(\mathbf{r}) \approx V(\mathbf{r}_0) + \|\nabla V\| dr$$

$$N = dr = \frac{T}{\|\nabla V\|} = \frac{T}{g}$$

Geoid heights changes

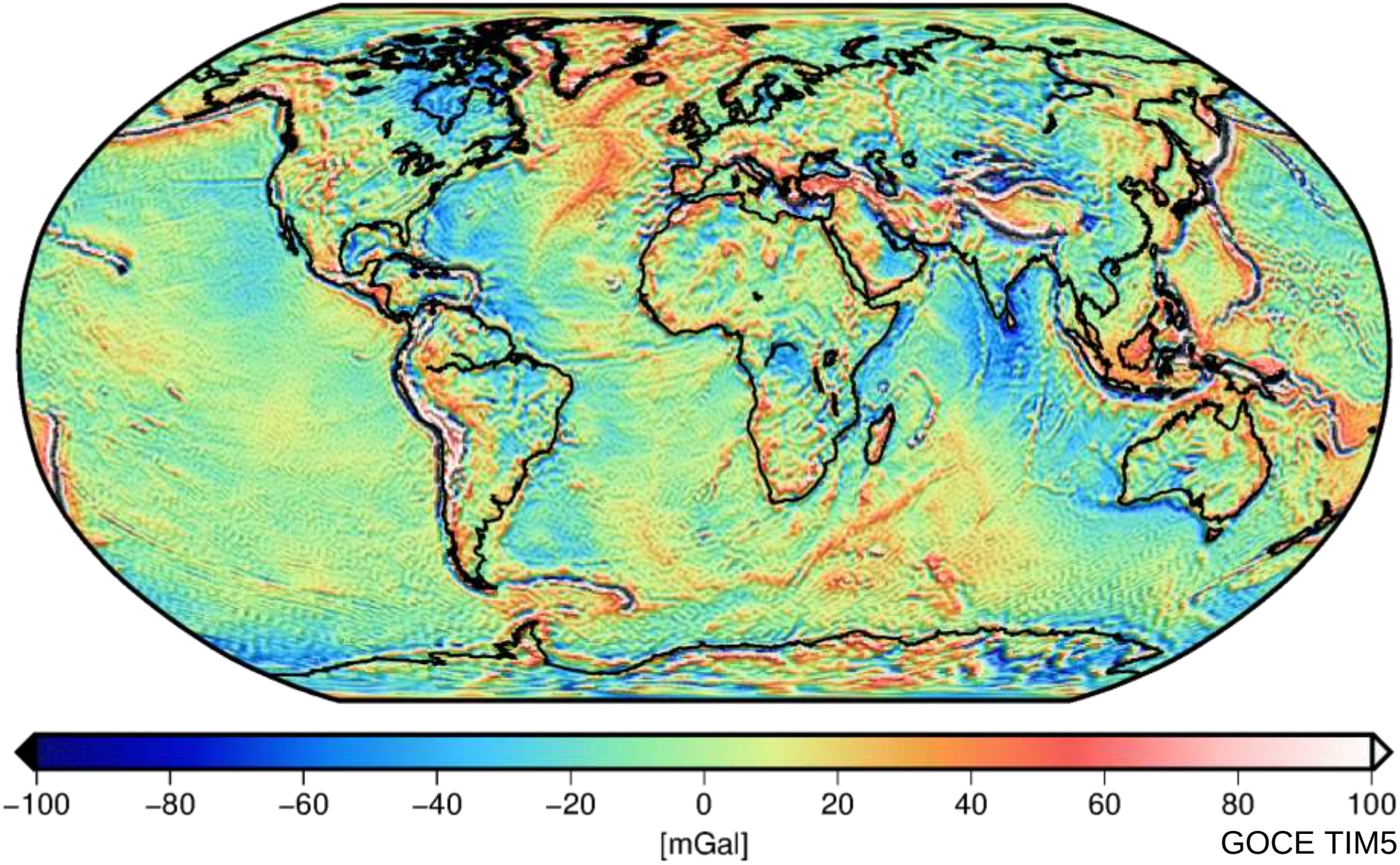
$$\Delta N = \frac{\Delta T}{g}$$

Spherical approx.

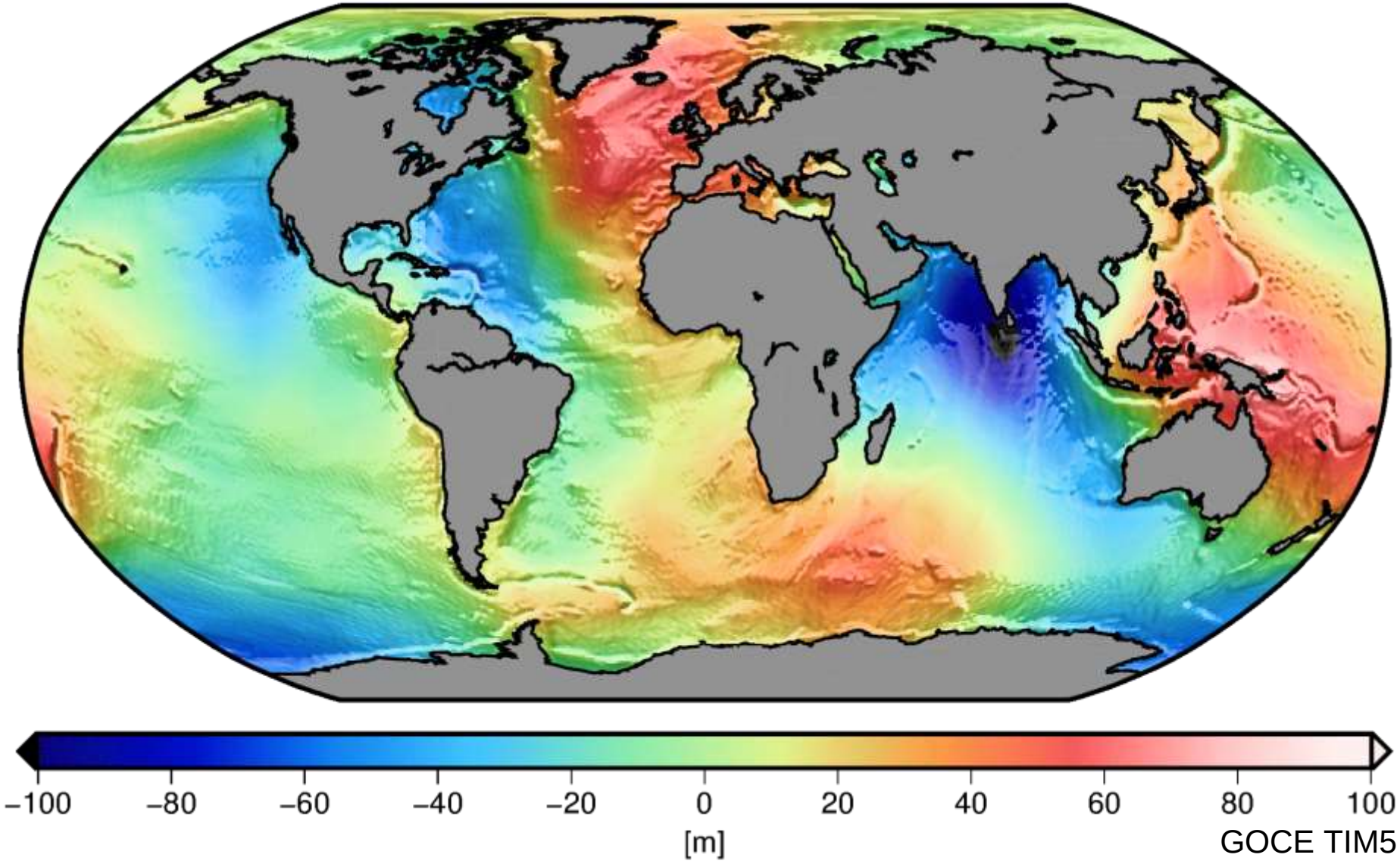
$$g \approx \frac{GM}{R^2}$$

$$\Delta N(\lambda,\vartheta) = R\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm}Y_{nm}(\lambda,\vartheta)$$

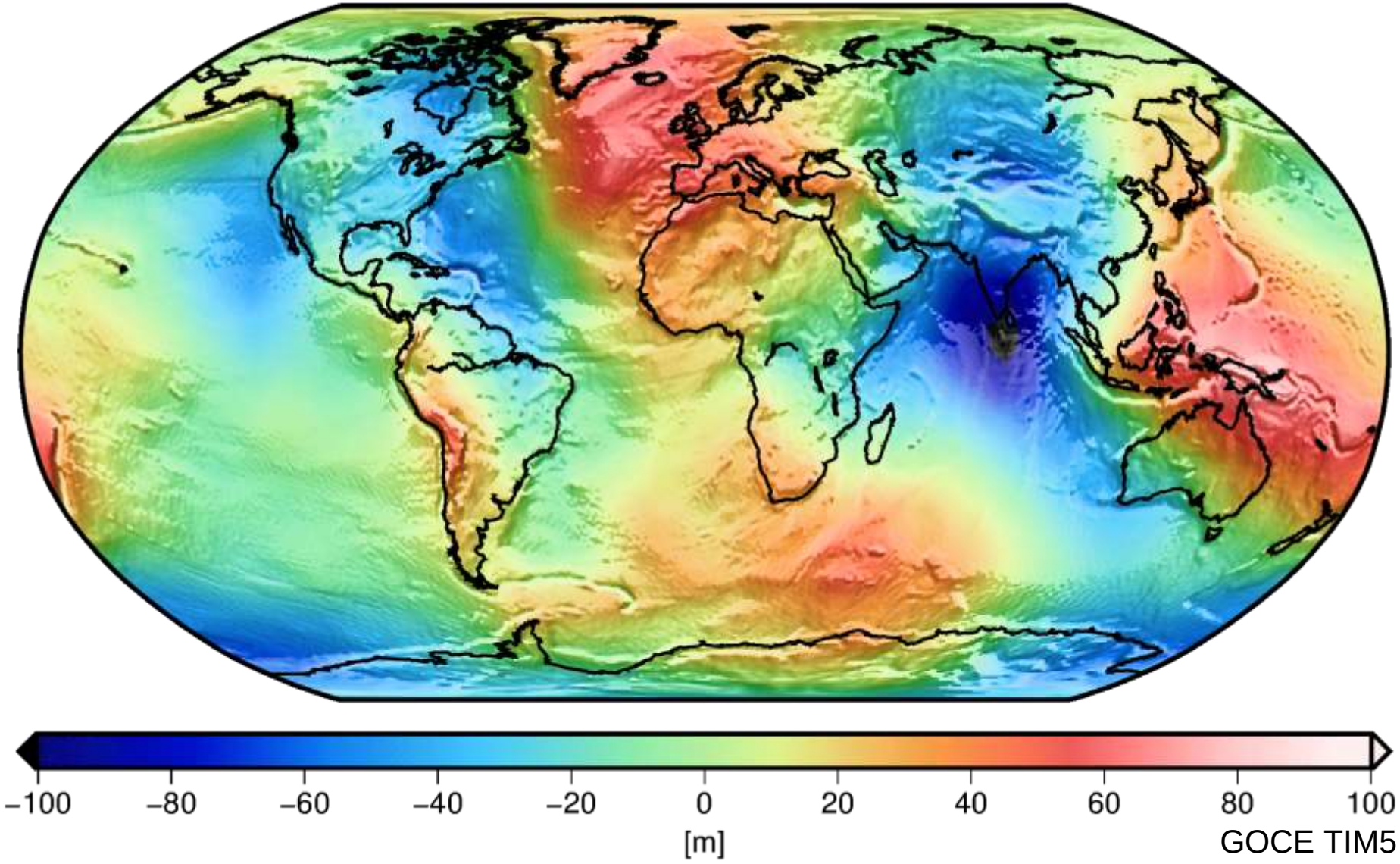
Gravity anomalies



Sea surface at rest



Geoid = equipotential surface



Gravity field functionals

Disturbing potential

$$T = V - U$$

Normal potential

$$T(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

gravity

$$g = \|\nabla V\|$$

Geoid heights

$$V(\mathbf{r}) \approx V(\mathbf{r}_0) + \|\nabla V\| dr$$

$$N = dr = \frac{T}{\|\nabla V\|} = \frac{T}{g}$$

Geoid heights changes

$$\Delta N = \frac{\Delta T}{g}$$

Spherical approx.

$$g \approx \frac{GM}{R^2}$$

$$\Delta N(\lambda,\vartheta) = R\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm}Y_{nm}(\lambda,\vartheta)$$

Gravity field functionals

Disturbing potential

$$T = V - U$$

Normal potential

$$T(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

Gravity disturbances

$$\delta g = \left\|\nabla T\right\| \approx -\frac{\partial T}{\partial r}$$

$$\delta g(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\frac{n+1}{r}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

Gravity anomlies

$$\Delta g = g(r_P) - \gamma(r_Q) \approx -\frac{\partial T}{\partial r} - 2\frac{T}{r}$$

$$\Delta g(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\frac{n-1}{r}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

Surface density changes

$$\Delta\sigma \approx -\frac{1}{4\pi RG}\left(2\frac{\partial \Delta T}{\partial r} + \frac{\Delta T}{r}\right)\Bigg|_{r=R}$$

$$\Delta\sigma(\lambda,\vartheta) = \frac{M}{4\pi R^2}\sum_{n=0}^{\infty}(2n+1)\sum_{m=-n}^{n}\Delta a_{nm}Y_{nm}(\lambda,\vartheta)$$

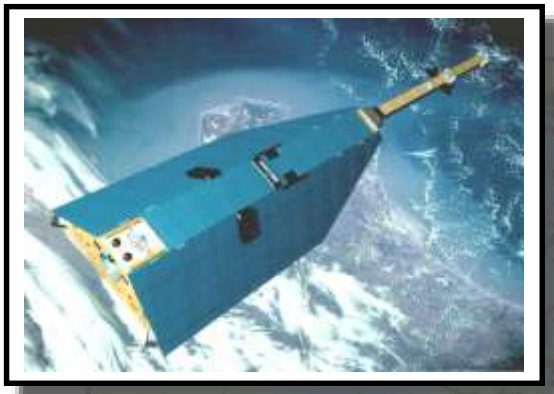
$$\rho_e = \frac{M}{4/3\,\pi R^3}$$

Total water storage changes

$$\Delta TWS(\lambda,\vartheta) = \frac{\rho_e R}{3}\sum_{n=0}^{\infty}\frac{2n+1}{1+k_n}\sum_{m=-n}^{n}\Delta a_{nm}Y_{nm}(\lambda,\vartheta)$$

Accuracy of gravity fields and Degree variances

ITSG-Champ2011 - Geoid



How accurate is the estimation of the geoid?

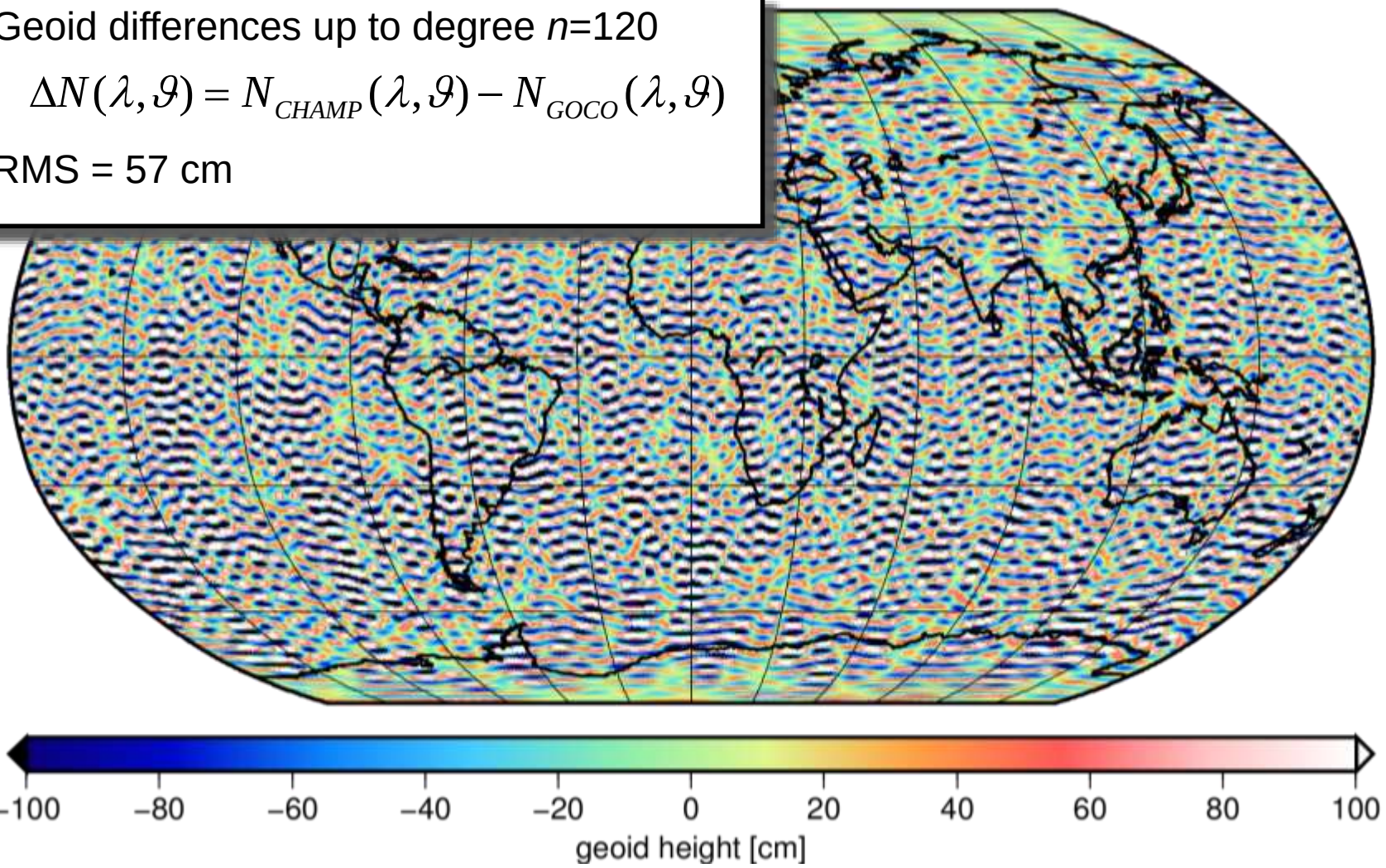
Comparison with a more accurate reference model

ITSG-Champ2011 – GOCO05s

Geoid differences up to degree *n*=120

$$\Delta N(\lambda,\vartheta) = N_{CHAMP}(\lambda,\vartheta) - N_{GOCO}(\lambda,\vartheta)$$

RMS = 57 cm

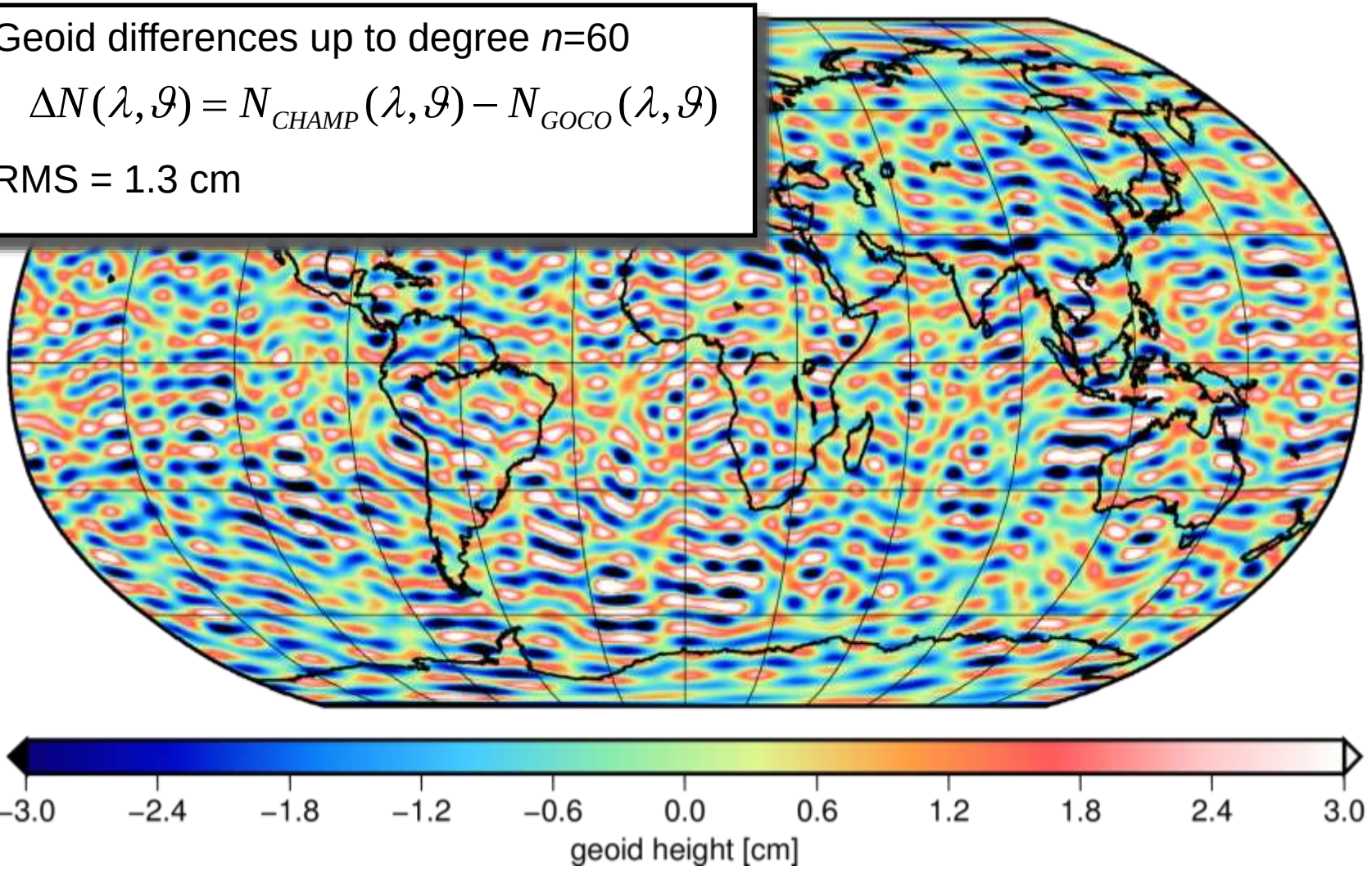


ITSG-Champ2011 – GOCO05s

Geoid differences up to degree *n*=60

$$\Delta N(\lambda,\vartheta) = N_{CHAMP}(\lambda,\vartheta) - N_{GOCO}(\lambda,\vartheta)$$

RMS = 1.3 cm



Variance

Geoid differences

$$\Delta N(\lambda,\vartheta) = N_{CHAMP}(\lambda,\vartheta) - N_{GOCO}(\lambda,\vartheta)$$

Variance (first try):

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} \Delta N^2(\lambda_i,\vartheta_i)$$

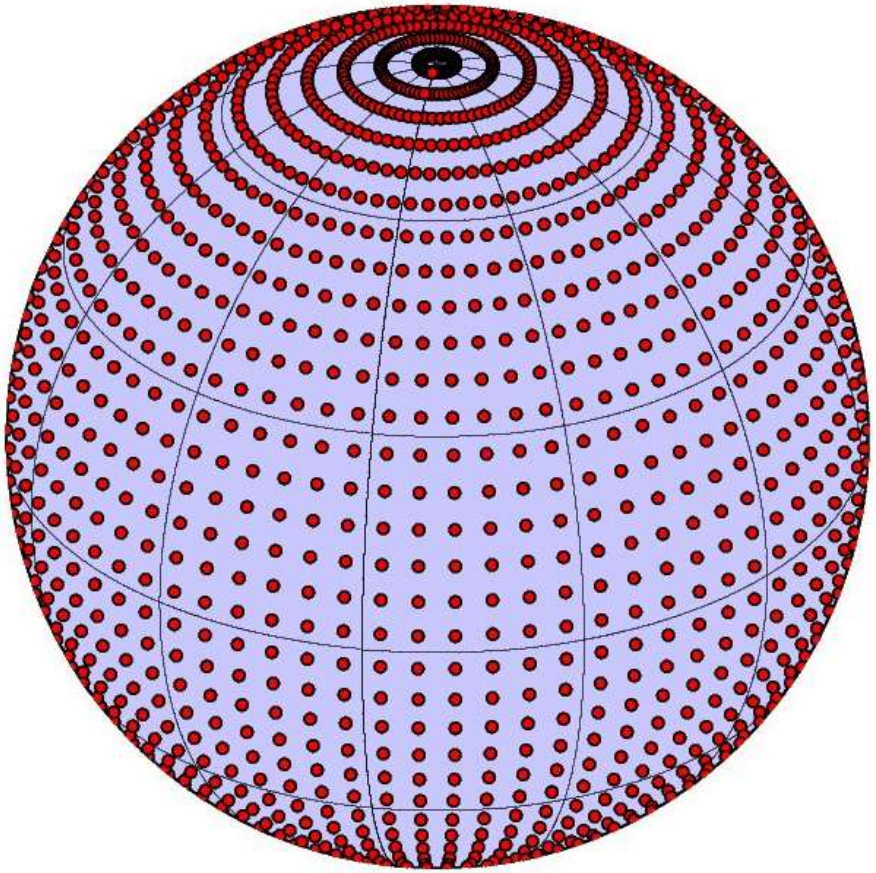
problem: a lot of values at the poles

with area weights:

$$\sigma^2 = \frac{1}{4\pi}\sum_{i=1}^{n} \Delta N^2(\lambda_i,\vartheta_i)\,\Delta\Phi_i$$

increase density of points to infinity
$\Rightarrow$ Integral:

$$\sigma^2 = \frac{1}{4\pi}\iint_{\Phi} \Delta N^2(\lambda,\vartheta)\,d\Phi$$



Variance

$$\sigma^2 = \frac{1}{4\pi} \iint_{\Phi} \Delta N^2(\lambda,\vartheta)\, d\Phi$$

Geoid differences

$$\Delta N(\lambda,\vartheta) = N_{CHAMP}(\lambda,\vartheta) - N_{GOCO}(\lambda,\vartheta)$$

$$= \frac{GM}{\gamma_0 R} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm}^{CHAMP} Y_{nm}(\lambda,\vartheta)$$

$$- \frac{GM}{\gamma_0 R} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm}^{GOCO} Y_{nm}(\lambda,\vartheta)$$

$$\gamma \approx \frac{GM}{R^2}$$

$$= \frac{GM}{\gamma_0 R} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Delta a_{nm} Y_{nm}(\lambda,\vartheta)$$

$$\approx R \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \Delta a_{nm} Y_{nm}(\lambda,\vartheta)$$

Variance

$$\sigma^2 = \frac{1}{4\pi}\iint_\Phi \Delta N^2(\lambda,\vartheta)\,d\Phi$$

$$= \frac{R^2}{4\pi}\iint_\Phi \left(\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm} Y_{nm}(\lambda,\vartheta)\right)^2 d\Phi$$

$$= \frac{R^2}{4\pi}\iint_\Phi \left(\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm} Y_{nm}(\lambda,\vartheta)\right)\left(\sum_{n'=0}^{\infty}\sum_{m'=-n}^{n'}\Delta a_{n'm'} Y_{n'm'}(\lambda,\vartheta)\right) d\Phi$$

$$= \frac{R^2}{4\pi}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\sum_{n'=0}^{\infty}\sum_{m'=-n}^{n'}\Delta a_{nm}\Delta a_{n'm'}\iint_\Phi Y_{n'm'}(\lambda,\vartheta) Y_{nm}(\lambda,\vartheta)\,d\Phi$$

$$= R^2\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm}^2$$

degree variance

$$\sigma_n^2 = \sum_{m=-n}^{n}\Delta a_{nm}^2$$

Geoid differences

$$\Delta N(\lambda,\vartheta) = N_{CHAMP}(\lambda,\vartheta) - N_{GOCO}(\lambda,\vartheta)$$

$$= \frac{GM}{\gamma_0 R}\sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_{nm}^{CHAMP} Y_{nm}(\lambda,\vartheta)$$

$$- \frac{GM}{\gamma_0 R}\sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_{nm}^{GOCO} Y_{nm}(\lambda,\vartheta)$$

$$= \frac{GM}{\gamma_0 R}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm} Y_{nm}(\lambda,\vartheta)$$

$$\approx R\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\Delta a_{nm} Y_{nm}(\lambda,\vartheta)$$

$$\gamma \approx \frac{GM}{R^2}$$

spherical harmonics are orthogonal

$$< Y_{nm}, Y_{n'm'} > =$$

$$= \iint_\Phi Y_{nm}(\lambda,\vartheta) Y_{n'm'}(\lambda,\vartheta)\,d\Phi$$

$$= 4\pi\delta_n^{n'}\delta_m^{m'}$$

Variance

$$\sigma^2 = \frac{1}{4\pi} \iint_{\Phi} \Delta N^2(\lambda, \vartheta)\, d\Phi$$

$$= R^2 \sum_{n=0}^{\infty} \Delta\sigma_n^2$$

with the degree variance $\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$

Root Mean Square (RMS)

$$\sigma = \sqrt{R^2 \sum_{n=0}^{\infty} \Delta\sigma_n^2}$$

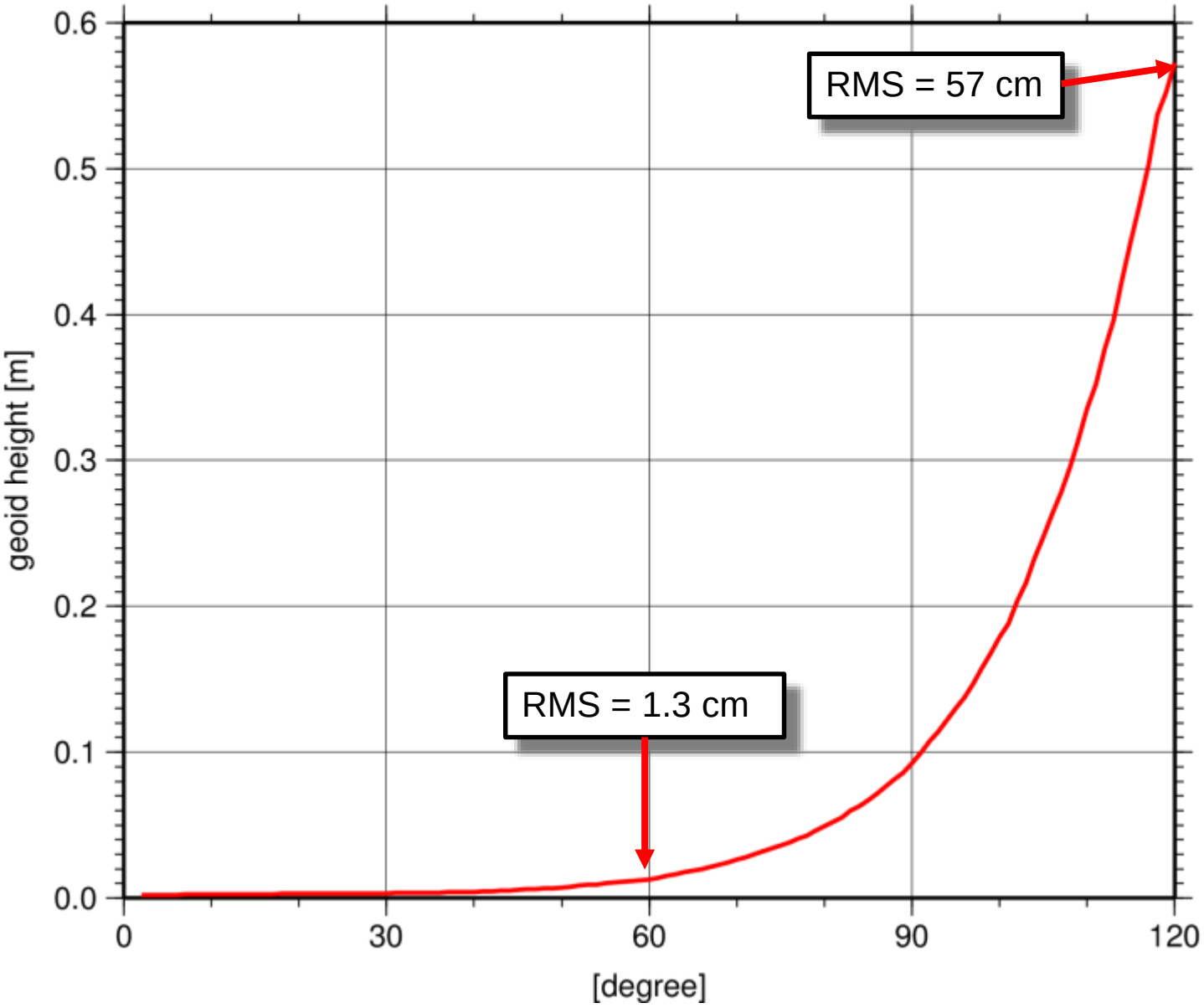
RMS (accumulated up to degree N)

RMS Geoid
up to degree *N*

$$\sigma = R\sqrt{\sum_{n=0}^{N}\Delta\sigma_n^2}$$

with

$$\Delta\sigma_n^2 = \sum_{m=-n}^{n}\Delta a_{nm}^2$$



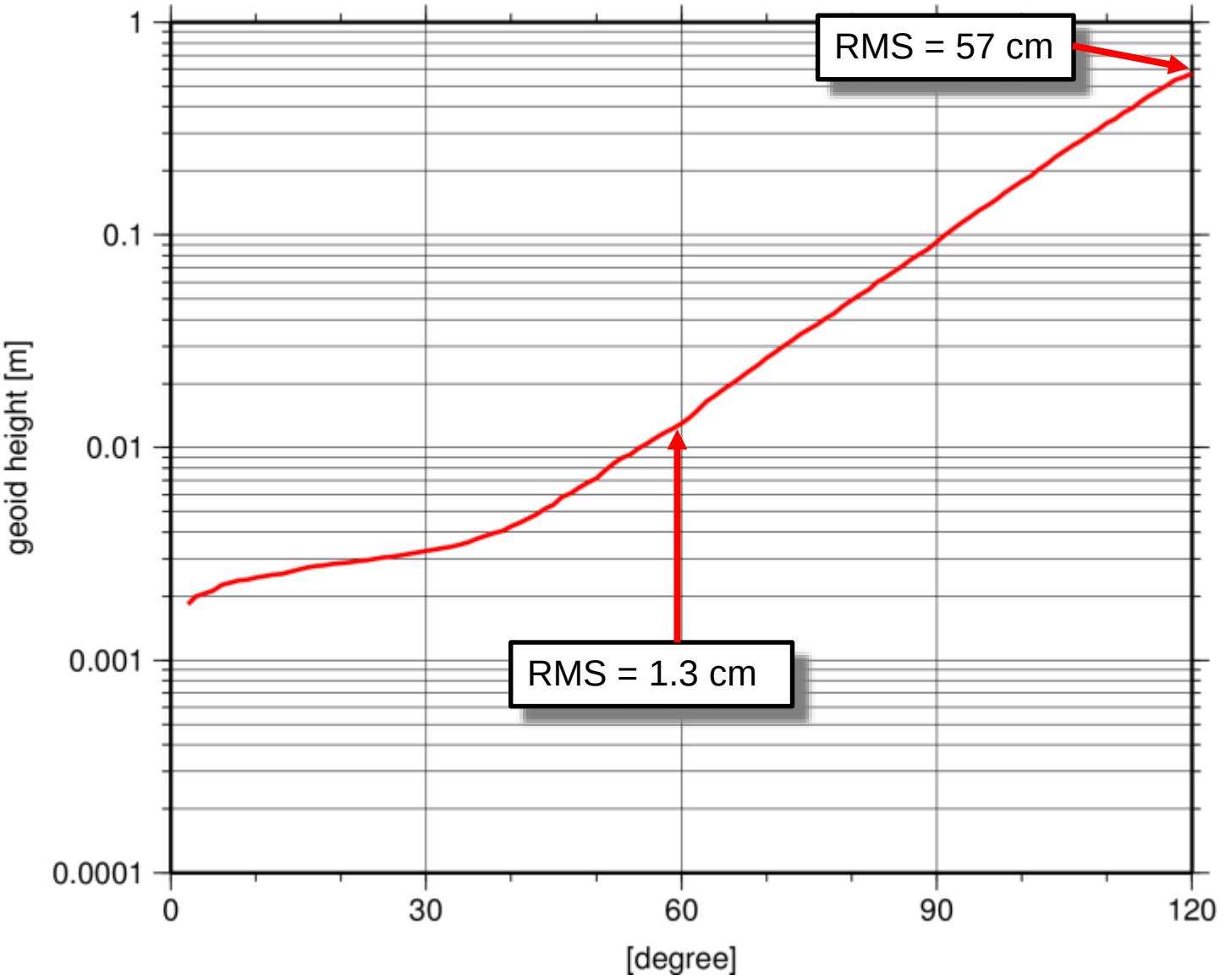
RMS (accumulated up to degree N)

RMS Geoid
up to degree *N*

$$\sigma = R\sqrt{\sum_{n=0}^{N}\Delta\sigma_n^2}$$

with

$$\Delta\sigma_n^2 = \sum_{m=-n}^{n}\Delta a_{nm}^2$$



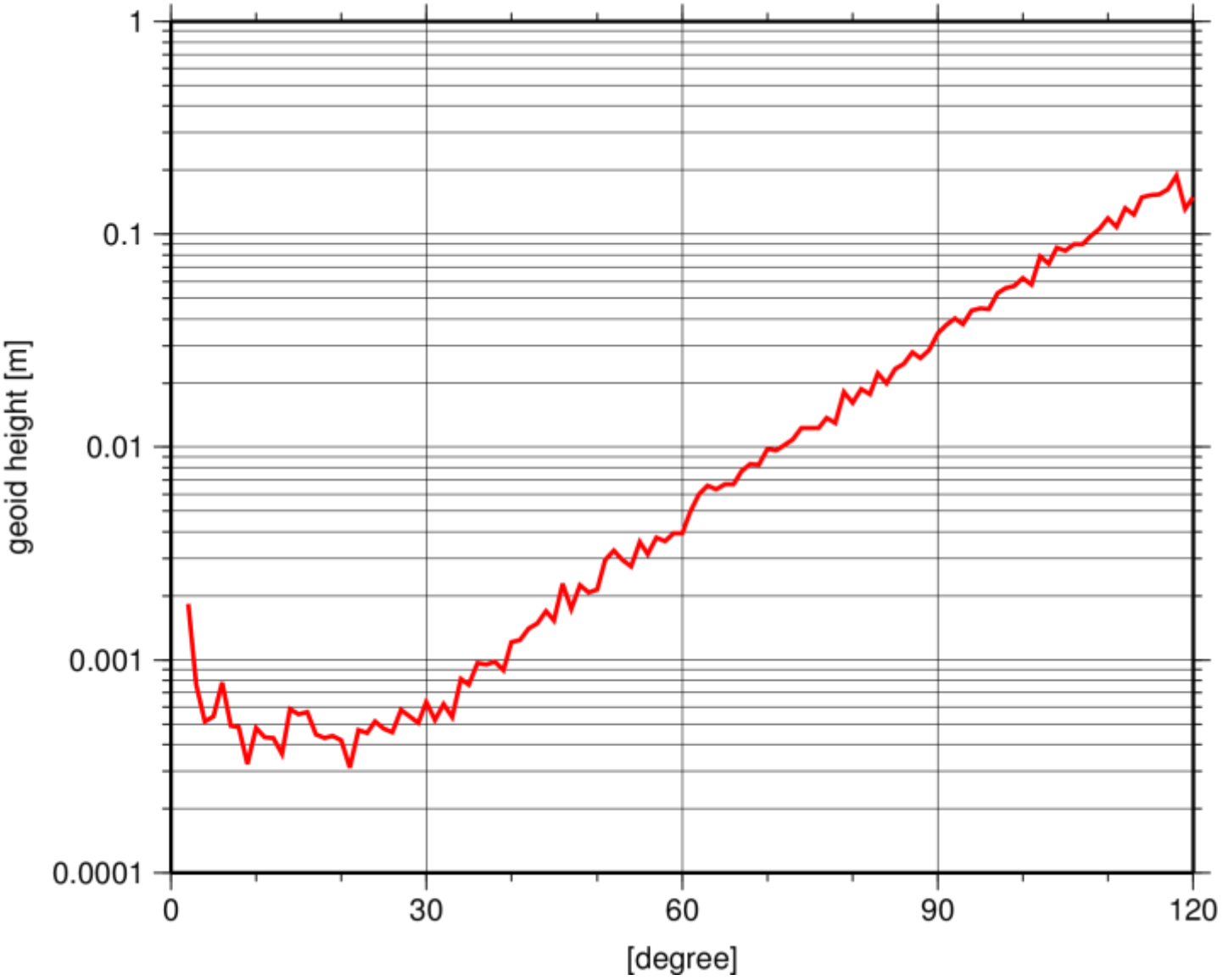
RMS (degree N)

RMS Geoid
of degree *N*

$$\sigma = R\sqrt{\Delta\sigma_n^2}$$

with

$$\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$$



RMS (degree N)

RMS Geoid
of degree *N*

$$\sigma = R\sqrt{\Delta\sigma_n^2}$$

with

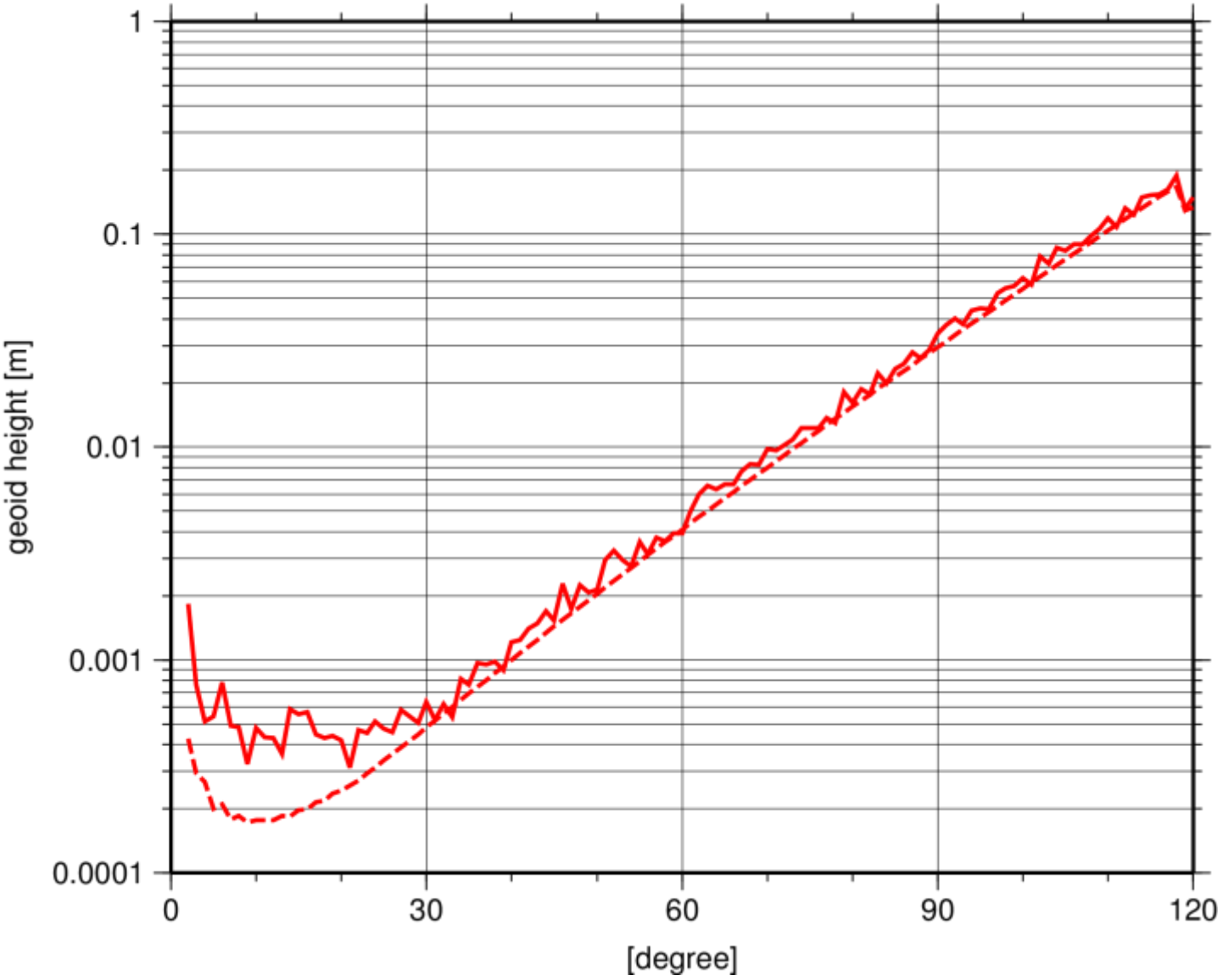
$$\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$$

Formal errors
(from adjustment)

$$\hat{\sigma} = R\sqrt{\hat{\sigma}_n^2}$$

with

$$\hat{\sigma}_n^2 = \sum_{m=-n}^{n} \hat{\sigma}^2(a_{nm})$$



Signal content

Variance of geoid differences

$$\sigma^2(\Delta N) = \frac{1}{4\pi}\iint_{\Phi} \Delta N^2(\lambda,\vartheta)\, d\Phi \quad = R^2 \sum_{n=0}^{\infty} \Delta\sigma_n^2$$

Difference degree variances

$$\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$$

Variance/Variability of geoid heights

$$\sigma^2(N) = \frac{1}{4\pi}\iint_{\Phi} N^2(\lambda,\vartheta)\, d\Phi \quad = R^2 \sum_{n=0}^{\infty} \sigma_n^2$$

Degree variances

$$\sigma_n^2 = \sum_{m=-n}^{n} a_{nm}^2$$

RMS (degree N)

RMS Geoid
of degree *N*

$$\sigma = R\sqrt{\Delta\sigma_n^2}$$

mit

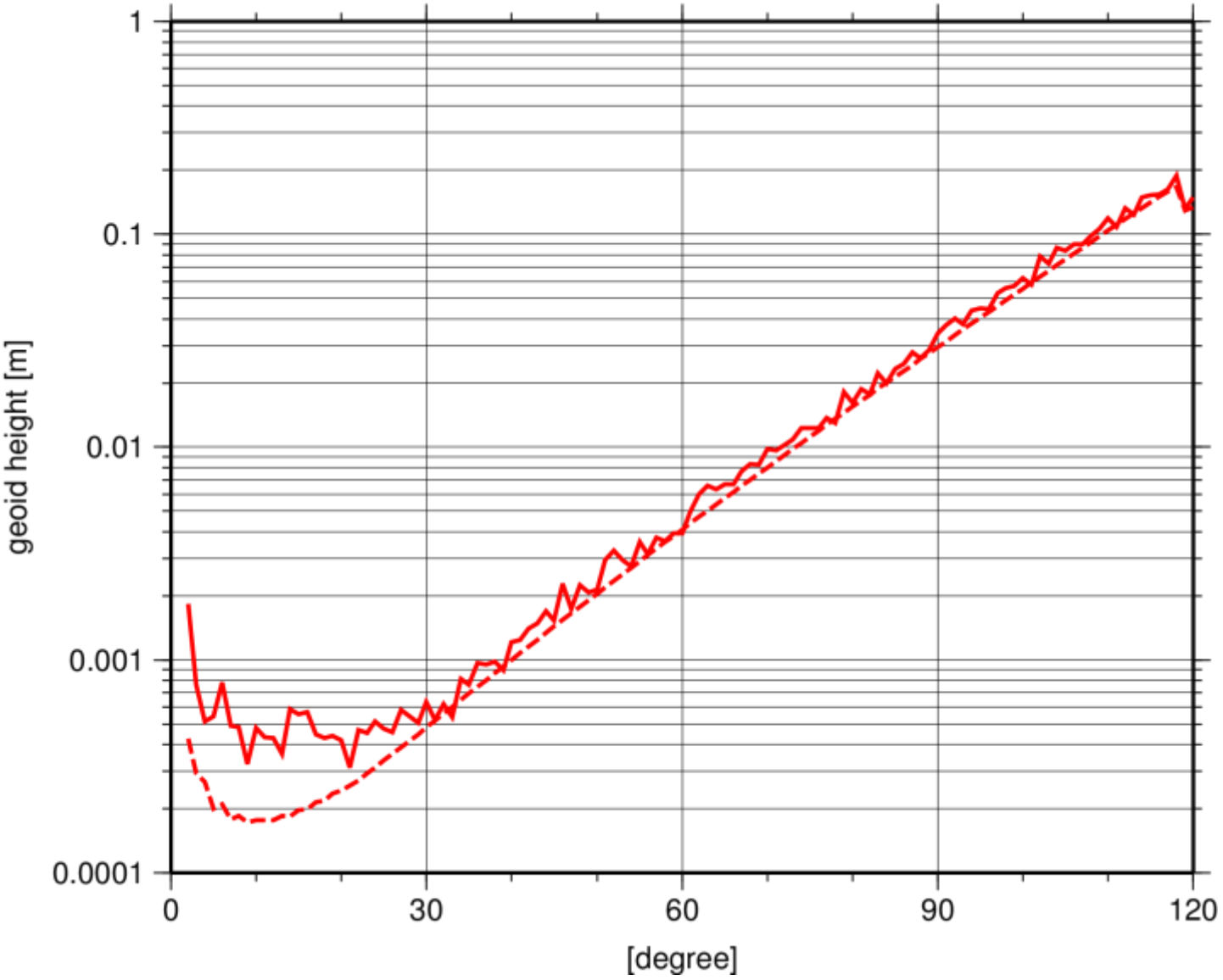
$$\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$$

Formal errors
(from adjustment)

$$\hat{\sigma} = R\sqrt{\hat{\sigma}_n^2}$$

with

$$\hat{\sigma}_n^2 = \sum_{m=-n}^{n} \hat{\sigma}^2(a_{nm})$$



RMS (degree N)

RMS Geoid of degree *N*

$$\sigma = R\sqrt{\Delta\sigma_n^2}$$

mit

$$\Delta\sigma_n^2 = \sum_{m=-n}^{n} \Delta a_{nm}^2$$

Formal errors (from adjustment)

$$\hat{\sigma} = R\sqrt{\hat{\sigma}_n^2}$$

with

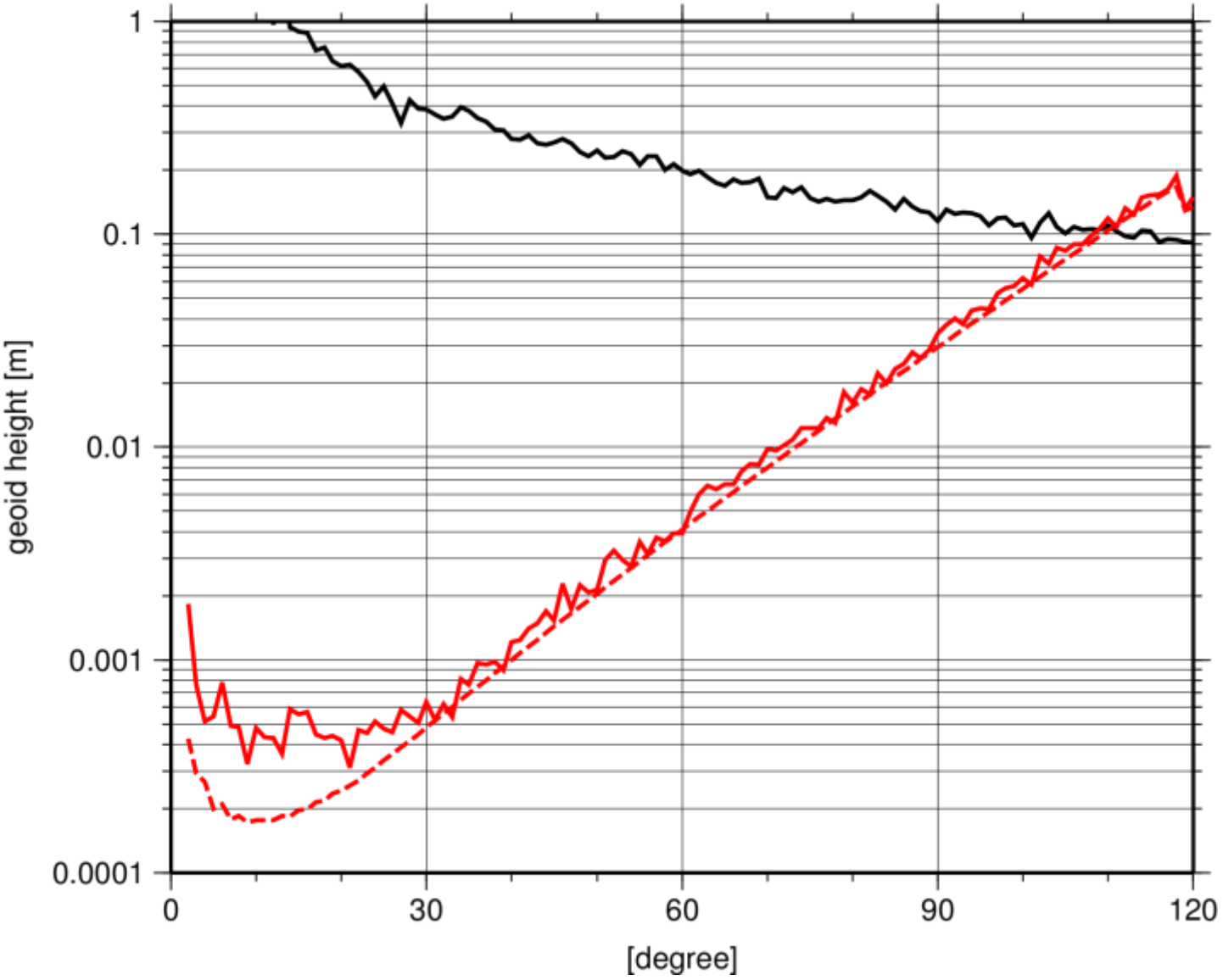
$$\hat{\sigma}_n^2 = \sum_{m=-n}^{n} \hat{\sigma}^2(a_{nm})$$

Signal

$$\sigma = R\sqrt{\sigma_n^2}$$

with

$$\sigma_n^2 = \sum_{m=-n}^{n} a_{nm}^2$$

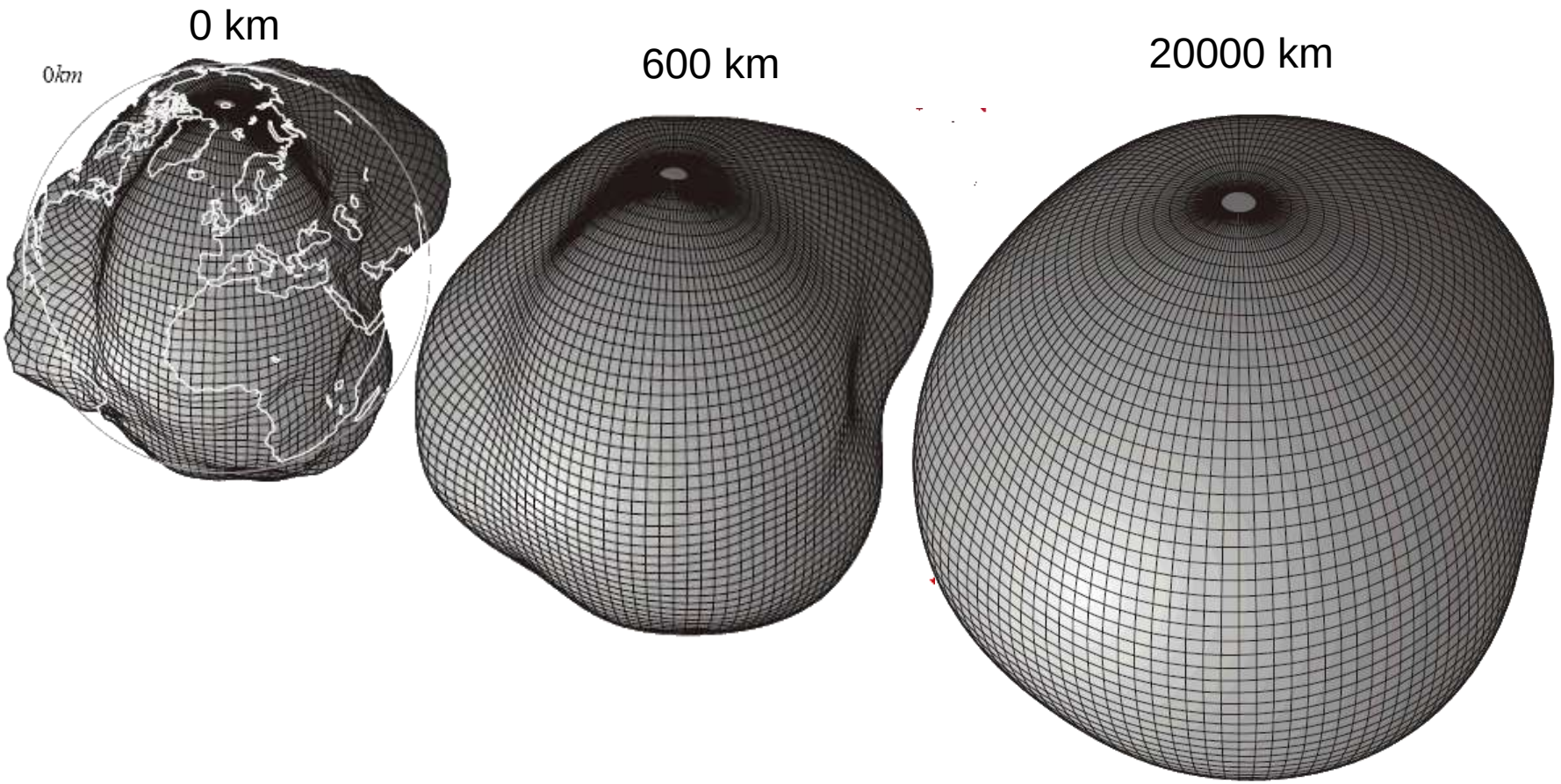


Upward continuation

Upward continuation

Gravitational potential

$$V(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$



Upward continuation

Gravitational potential

$$V(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} a_{nm}Y_{nm}(\lambda,\vartheta)$$

GRACE

$R = 6378\,km$

$r = R + 450\,km$

Damping factors

$\left(\frac{R}{r}\right)^{n+1} = 0{,}934095 \quad \text{für } n = 0$

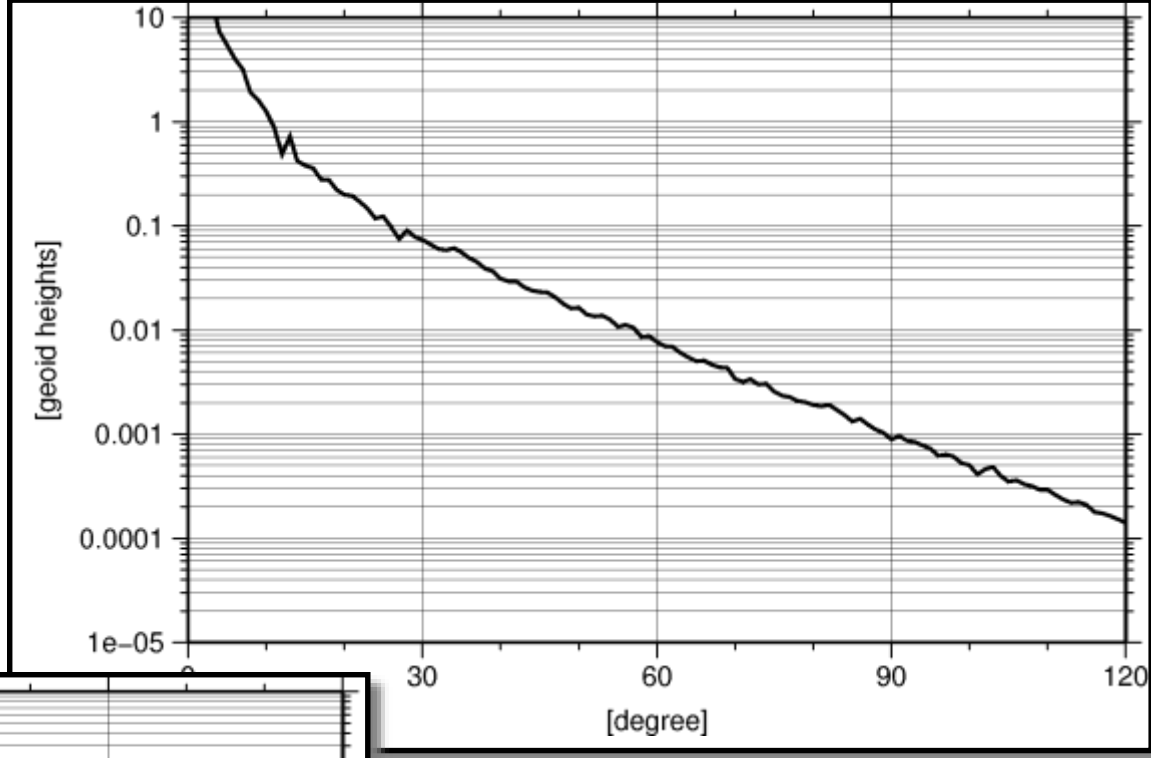
$\left(\frac{R}{r}\right)^{n+1} = 0.815029 \quad \text{für } n = 2 \quad (10.000\ \text{km})$

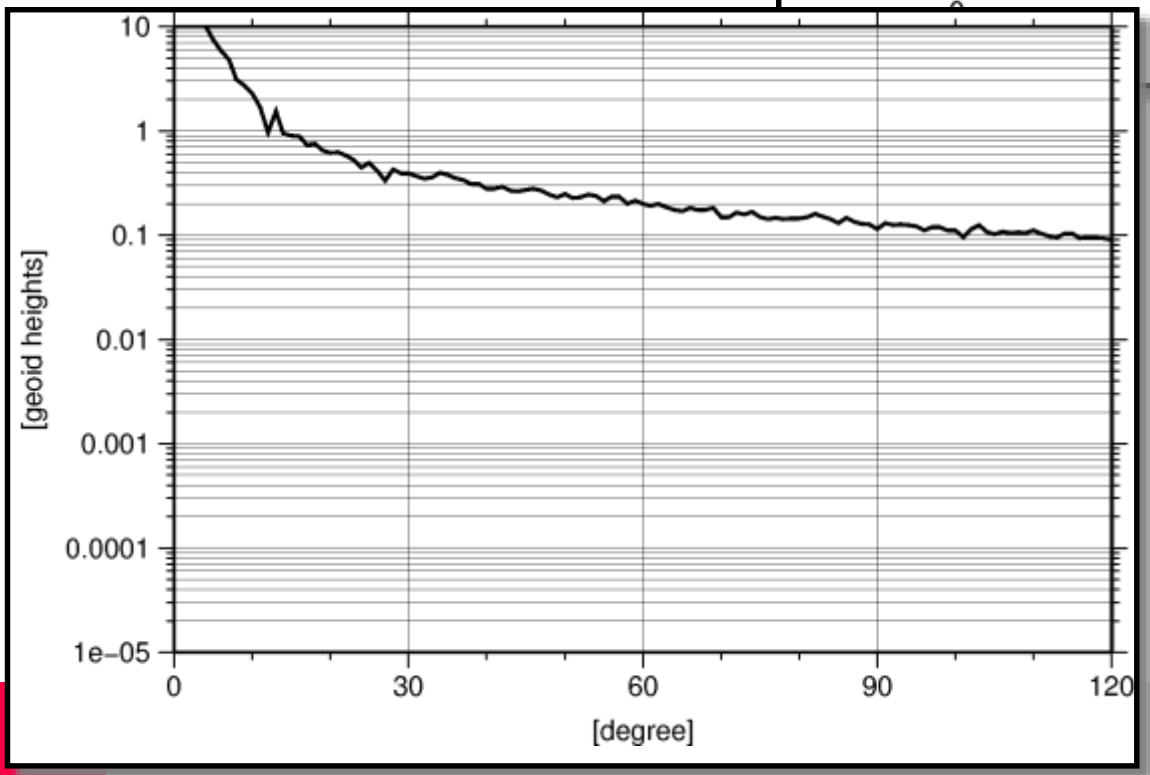
$\left(\frac{R}{r}\right)^{n+1} = 0.000261 \quad \text{für } n = 120 \quad (160\ \text{km})$

$\left(\frac{R}{r}\right)^{n+1} = 0.000004 \quad \text{für } n = 180 \quad (110\ \text{km})$

Degree variances

Satellite height (450 km):

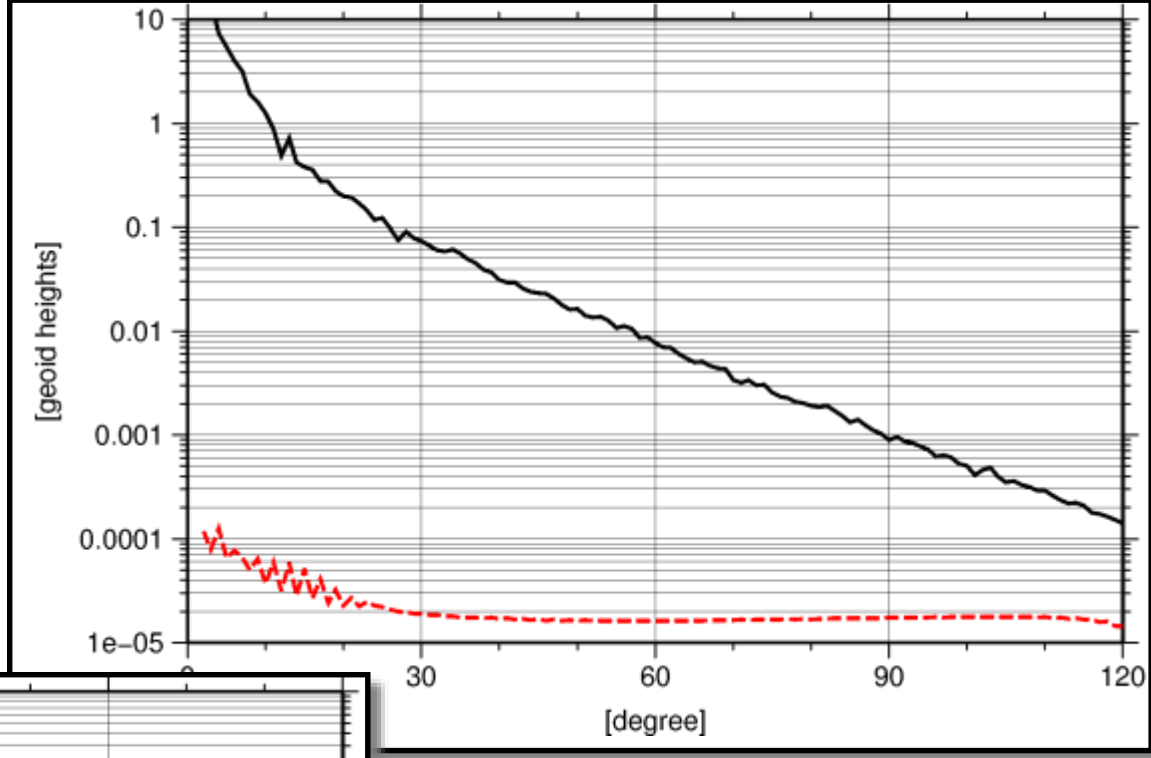


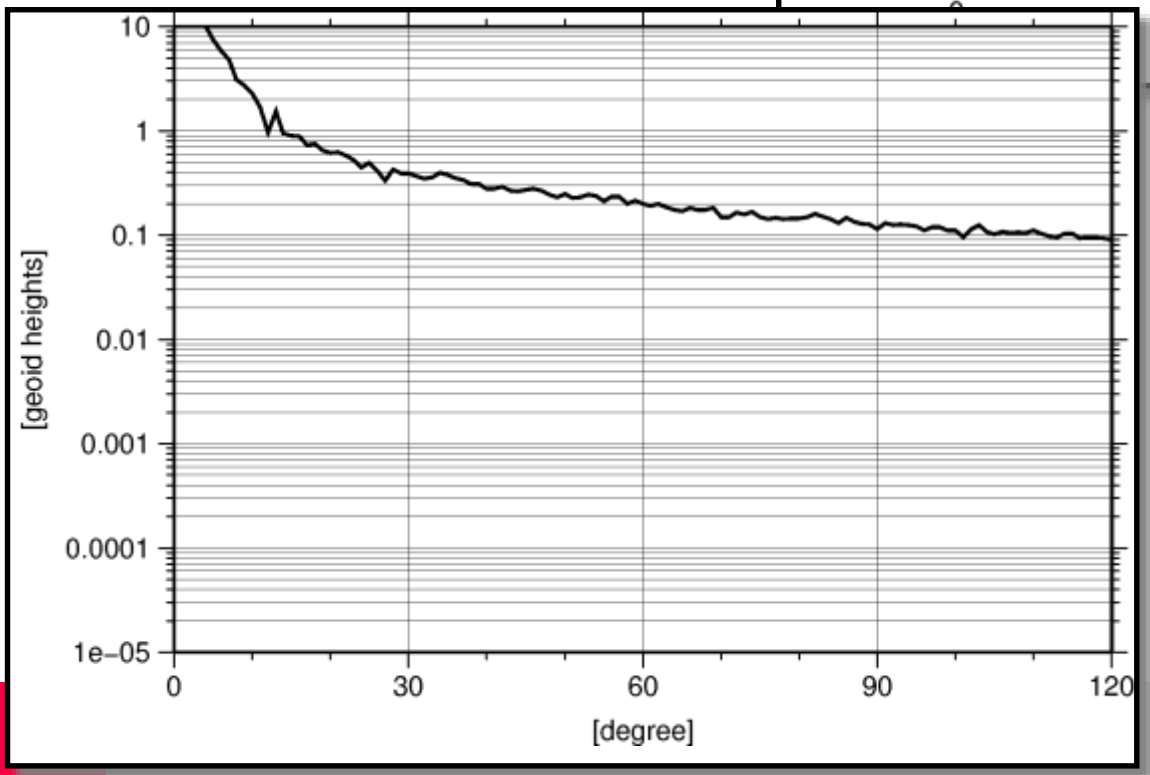


: Earth surface (0 km)

Degree variances

Satellite height (450 km):

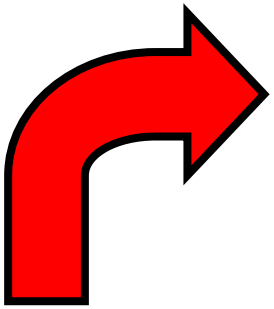


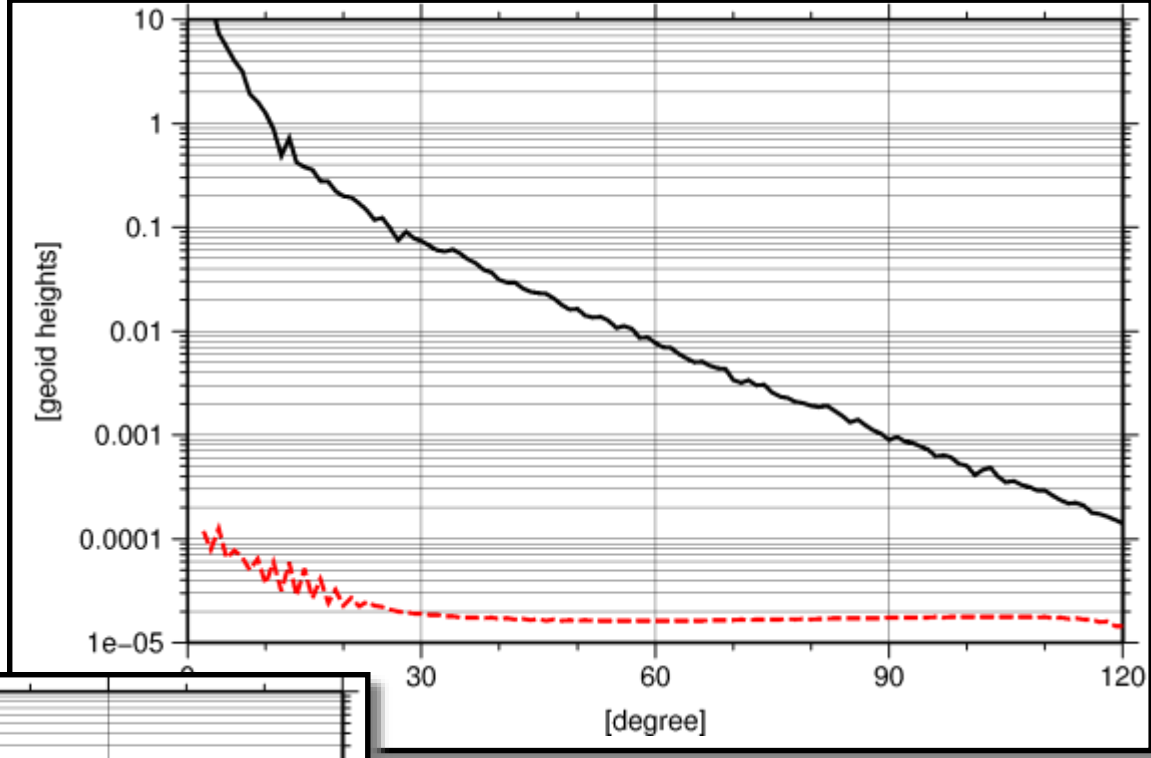


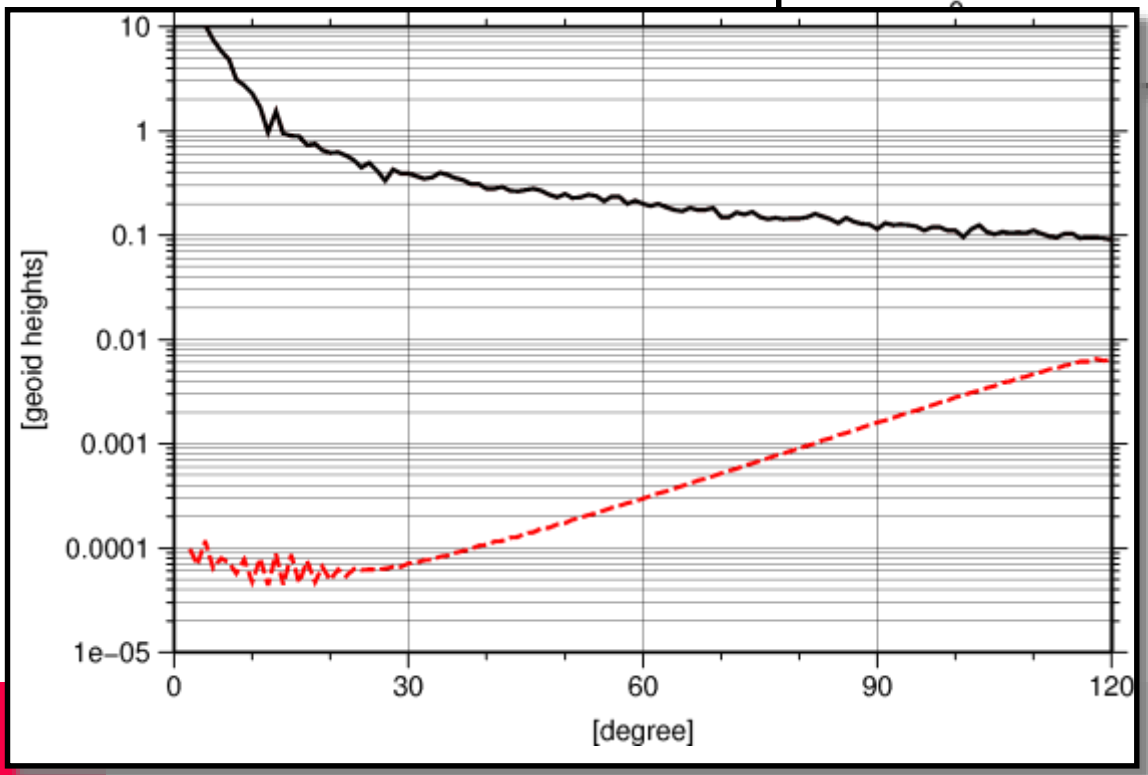
: Earth surface (0 km)

Degree variances

Satellite height (450 km):

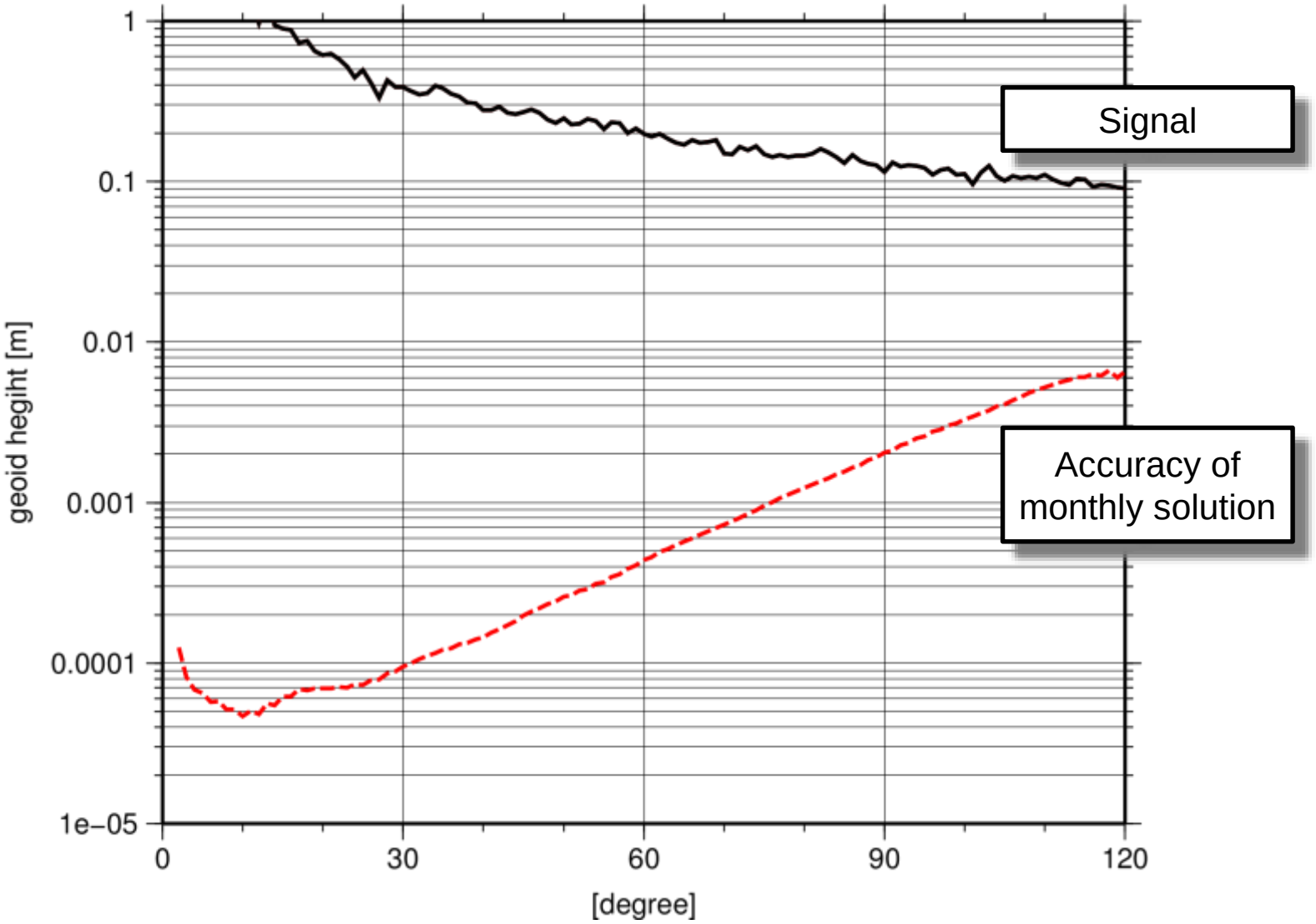




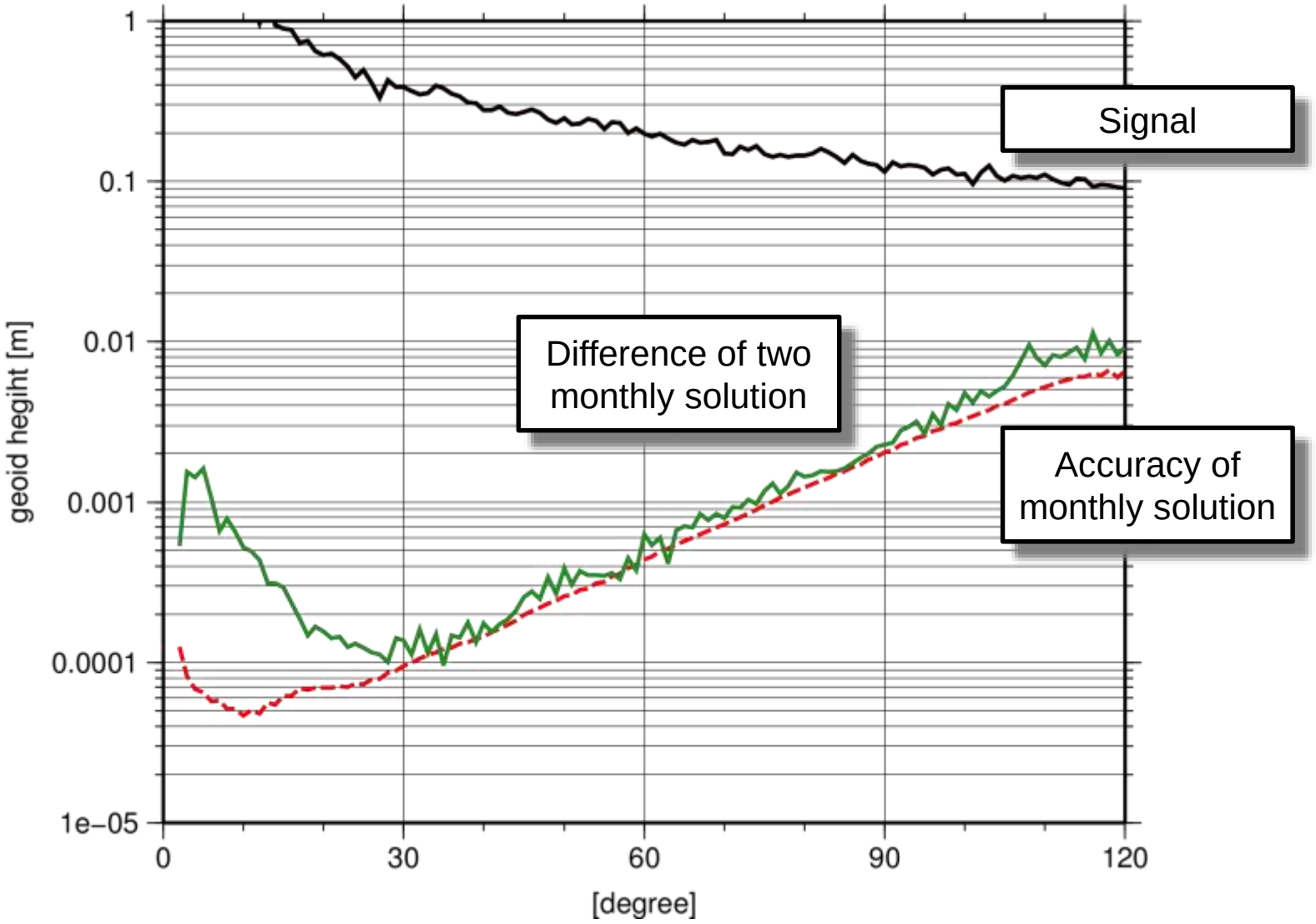


: Earth surface (0 km)

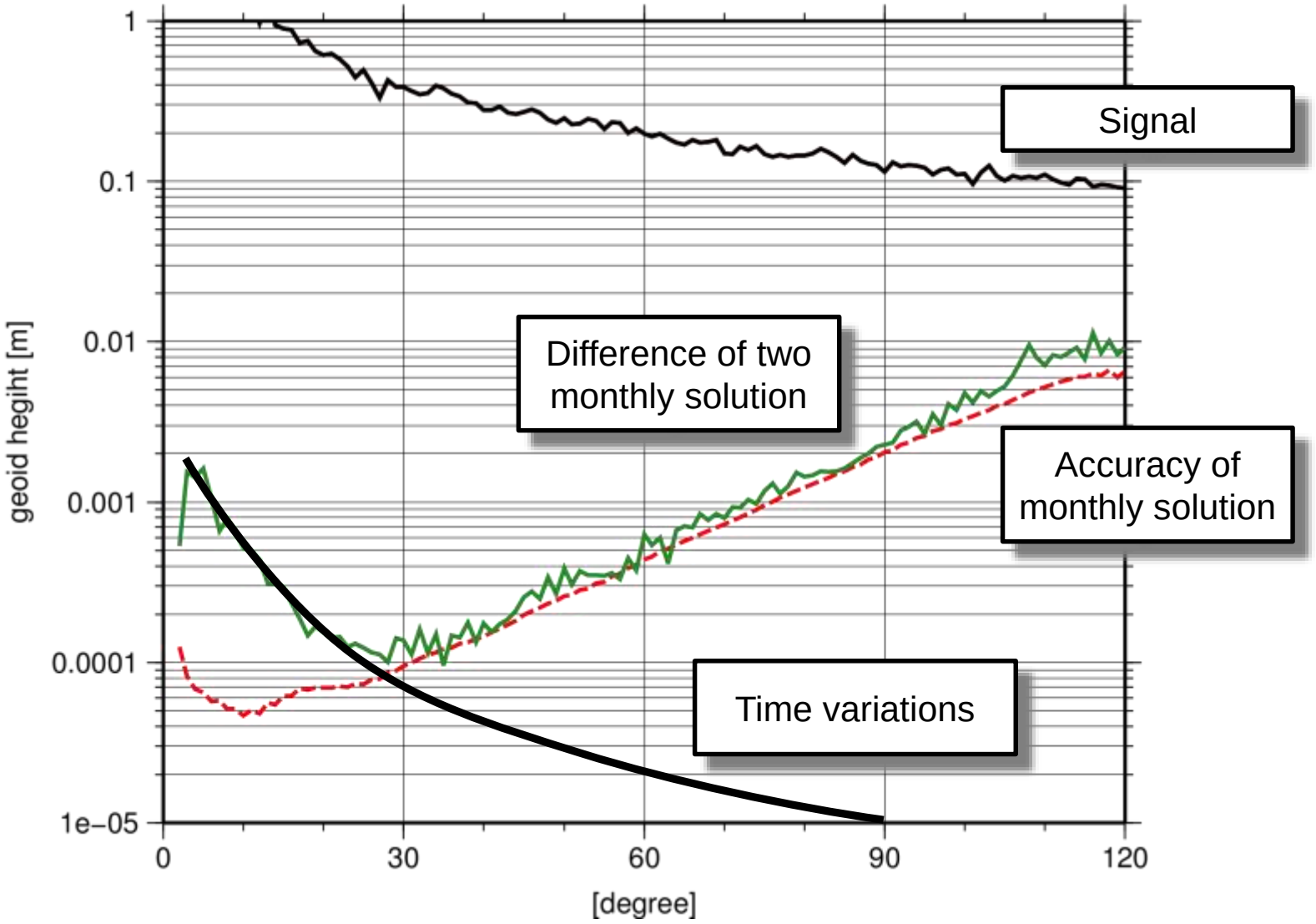
Degree variances



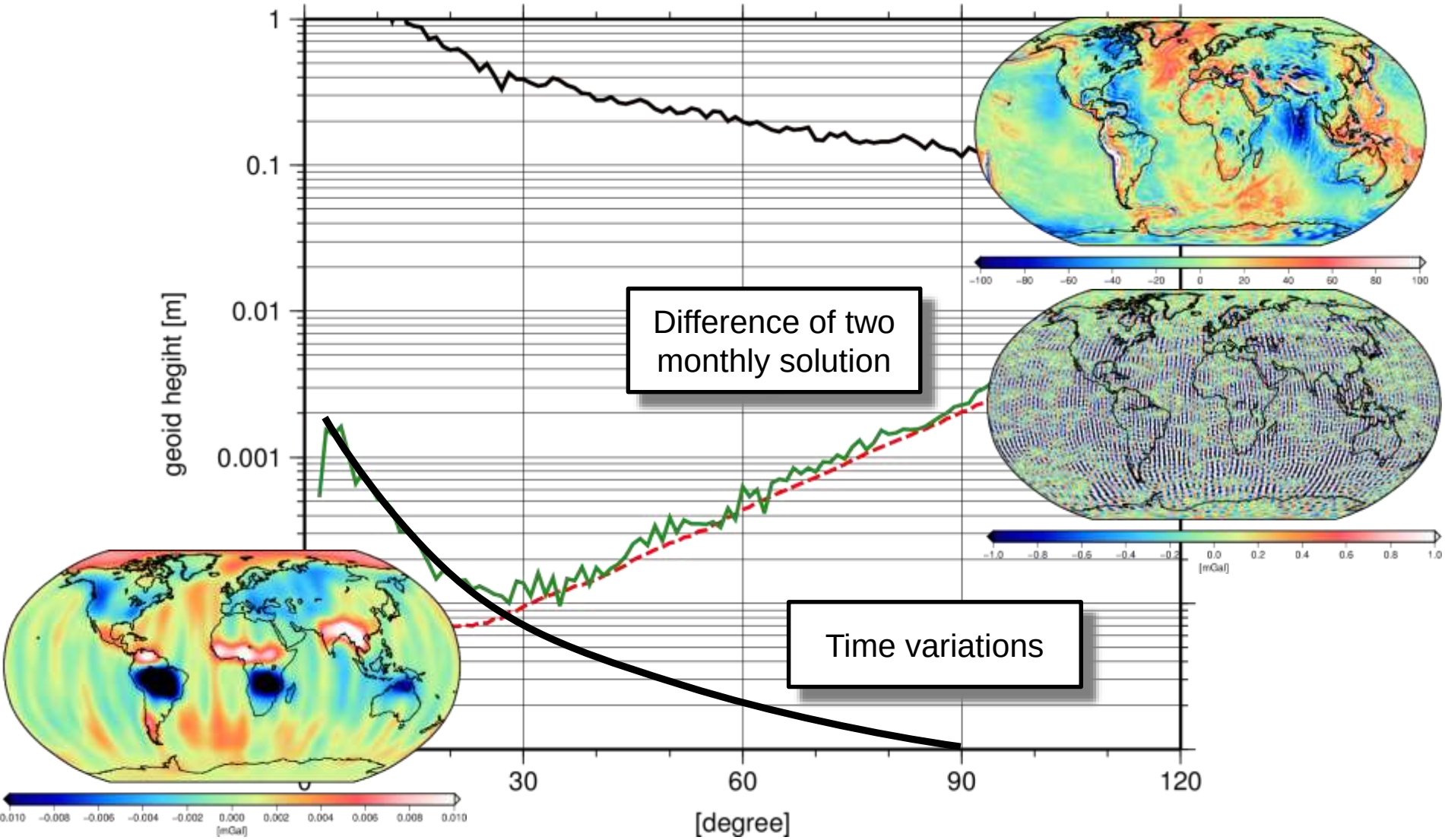
Degree variances



Degree variances



Degree variances

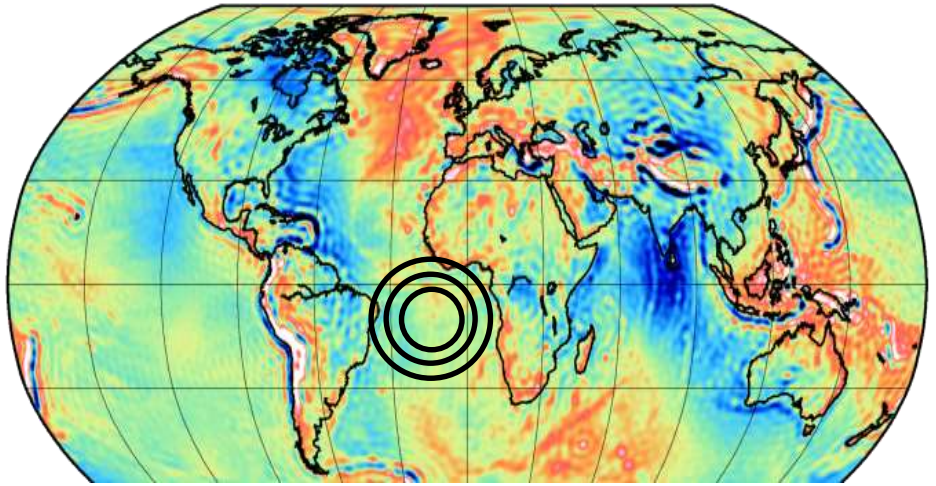


Filtering

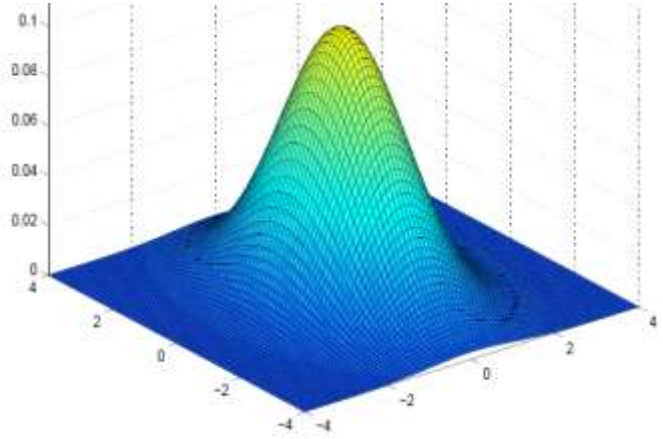
Filtering

Filtering the gravity field (Convolution)

$$V_F(\mathbf{x}) = \frac{1}{4\pi} \iint_{\Phi} F(\mathbf{x},\mathbf{y})\, V(\mathbf{y})\, d\Phi(\mathbf{y})$$



Gaussian Filter *F*



Gaussian filter

$$F(\cos\psi) = \frac{2b}{1-e^{-2b}} e^{-b(1-\cos\psi)}$$

with

$$b = \frac{\ln(2)}{1-\cos(\frac{d}{R})}$$

with *d*: filter radius

Degree variances

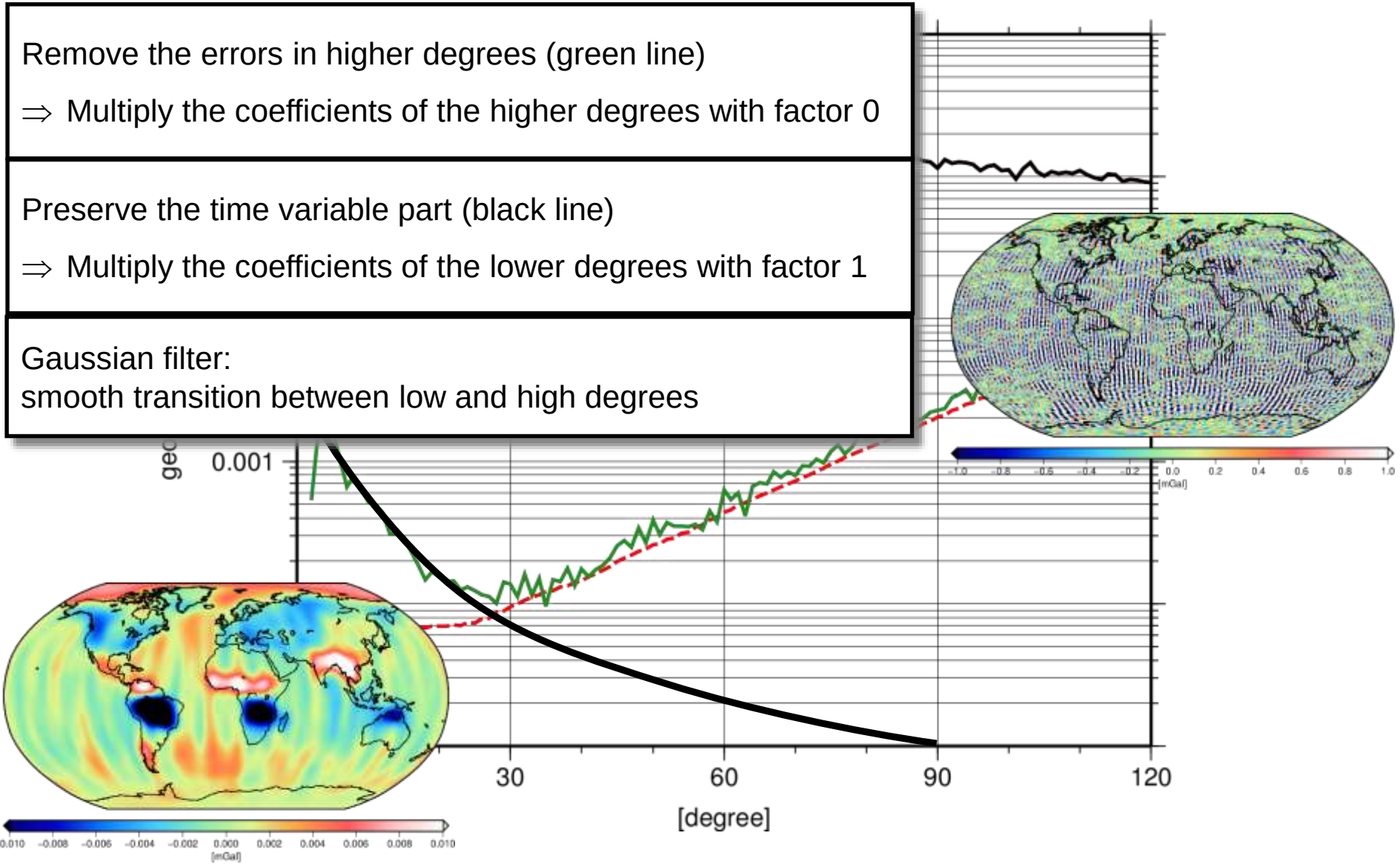
Remove the errors in higher degrees (green line)

⇒ Multiply the coefficients of the higher degrees with factor 0

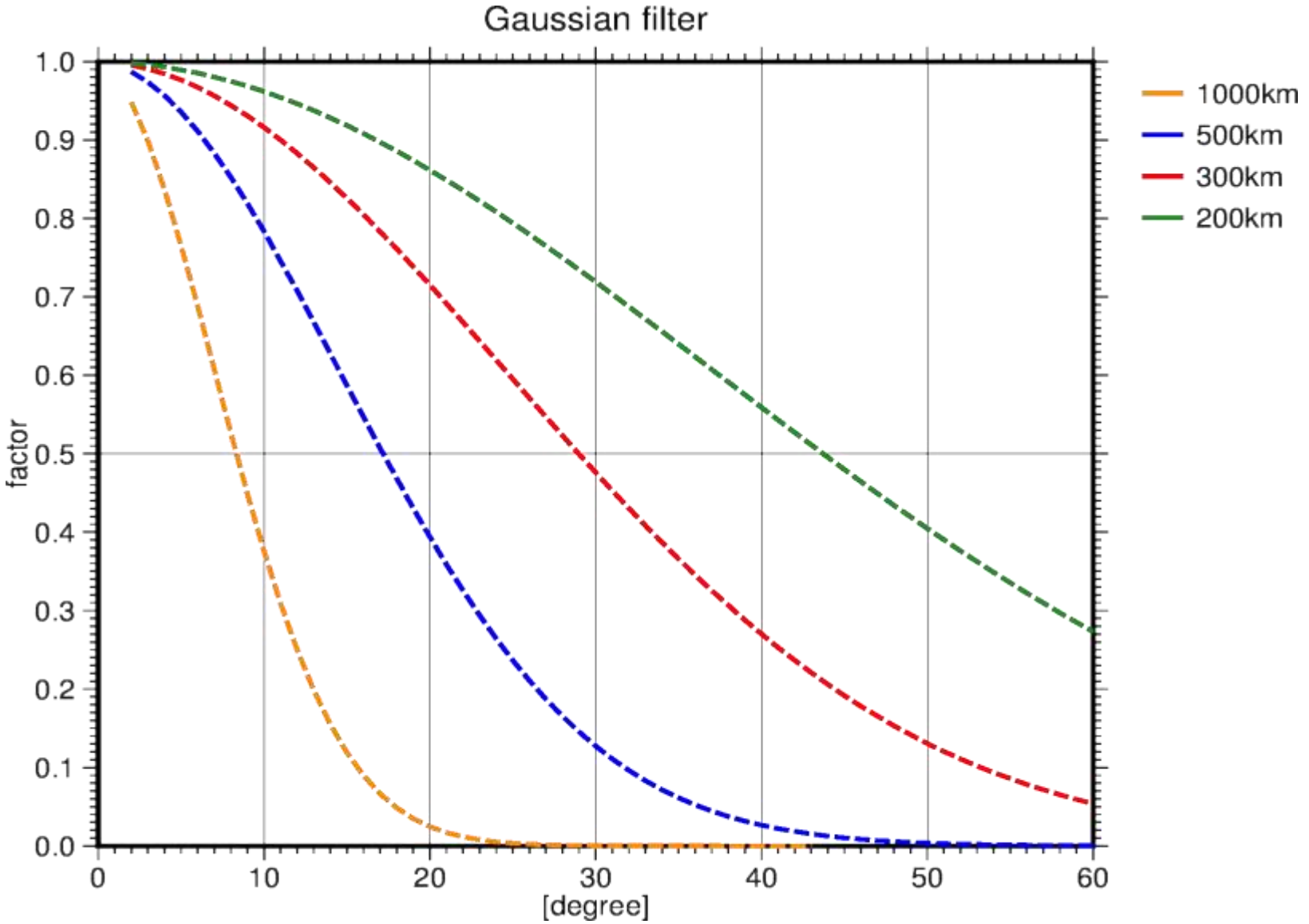
Preserve the time variable part (black line)

⇒ Multiply the coefficients of the lower degrees with factor 1

Gaussian filter:
smooth transition between low and high degrees



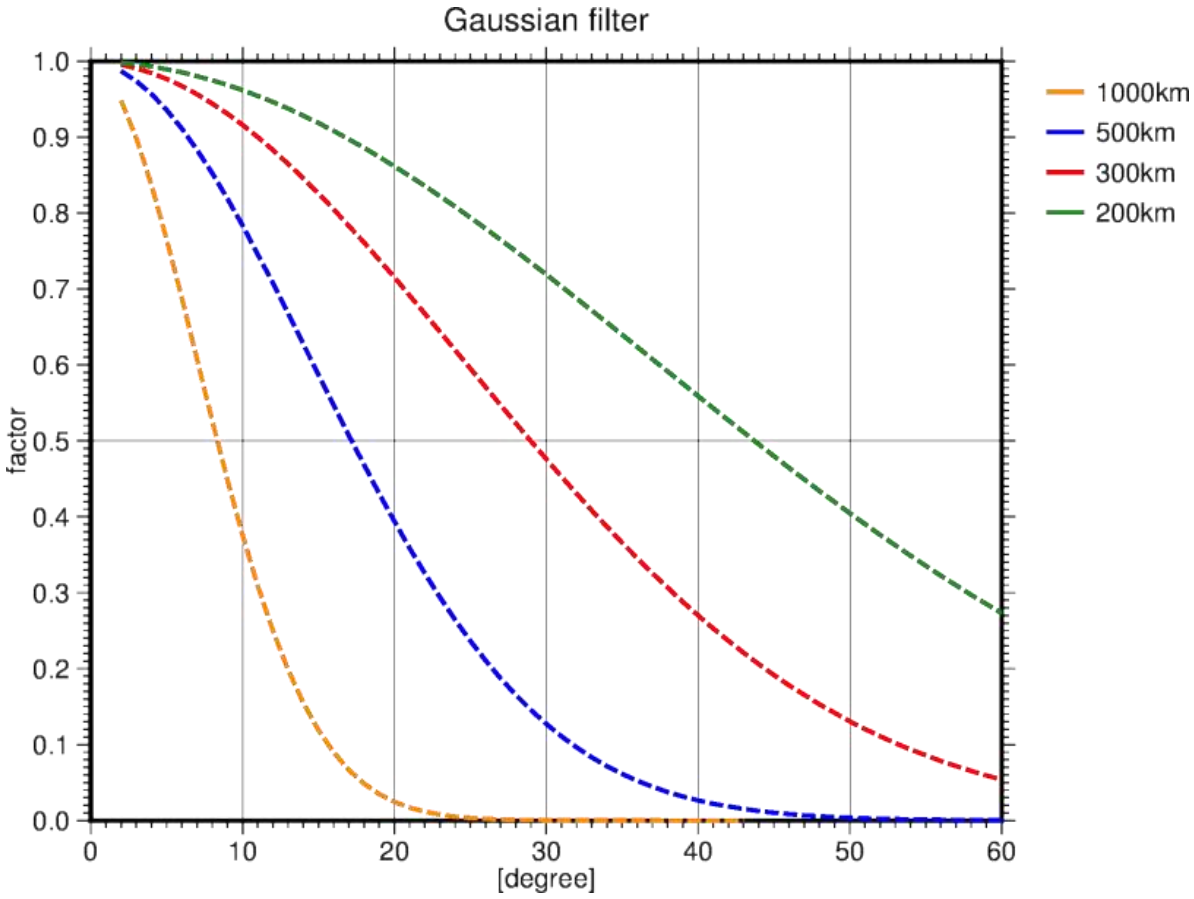
Gaussian filter



Gaussian filter

Filtered potential

$$V_F(\lambda,\vartheta,r) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1} w_n \sum_{m=-n}^{n} c_{nm}C_{nm}(\lambda,\vartheta) + s_{nm}S_{nm}(\lambda,\vartheta)$$



Gaussian filter coefficients

$$w_0 = 1$$

$$w_1 = \frac{1+e^{-2b}}{1-e^{-2b}} - \frac{1}{b}$$

$$w_n = -\frac{2n-1}{b} w_{n-1} + w_{n-2}$$

with

$$b = \frac{\ln(2)}{1-\cos(\frac{d}{R})}$$

with *d*: filter radius

200 km … 1000 km

Filtering

Unfiltered potential

$$V(\mathbf{y}) = \frac{GM}{R} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} a_{nm} Y_{nm}(\mathbf{y})$$

Filtering in the spatial domain

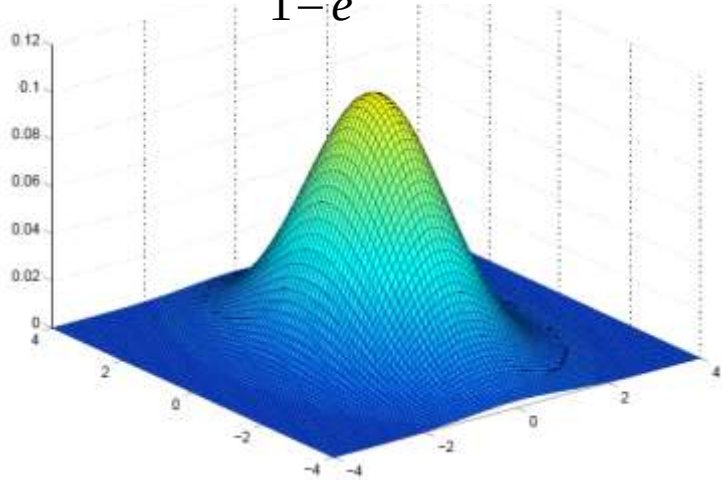
$$V_F(\mathbf{x}) = \frac{1}{4\pi} \iint_{\Phi} F(\mathbf{x},\mathbf{y})\, V(\mathbf{y})\, d\Phi(\mathbf{y})$$

Filtering in the spectral domain

$$V_F(\mathbf{x}) = \frac{GM}{R} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} w_n a_{nm} Y_{nm}(\mathbf{x})$$

Gaussian Filter

$$F(\cos\psi) = \frac{2b}{1-e^{-2b}} e^{-b(1-\cos\psi)}$$

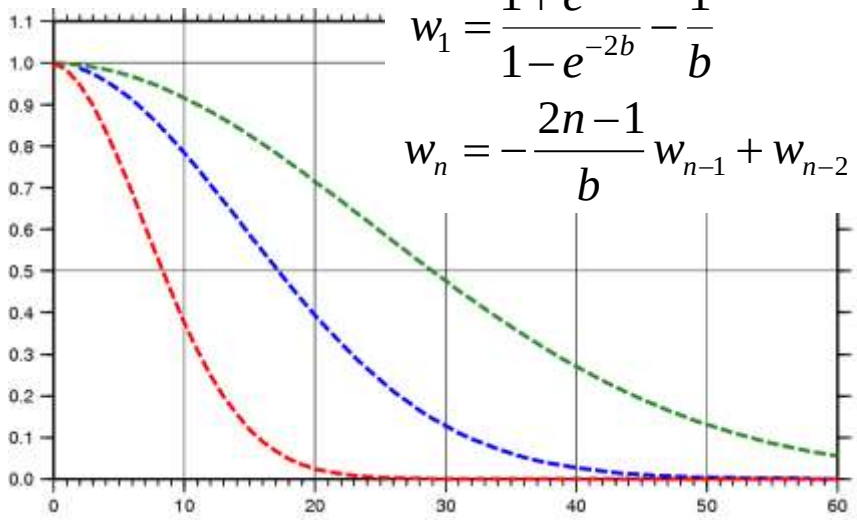


Filter coefficients

$$w_0 = 1$$

$$w_1 = \frac{1+e^{-2b}}{1-e^{-2b}} - \frac{1}{b}$$

$$w_n = -\frac{2n-1}{b} w_{n-1} + w_{n-2}$$



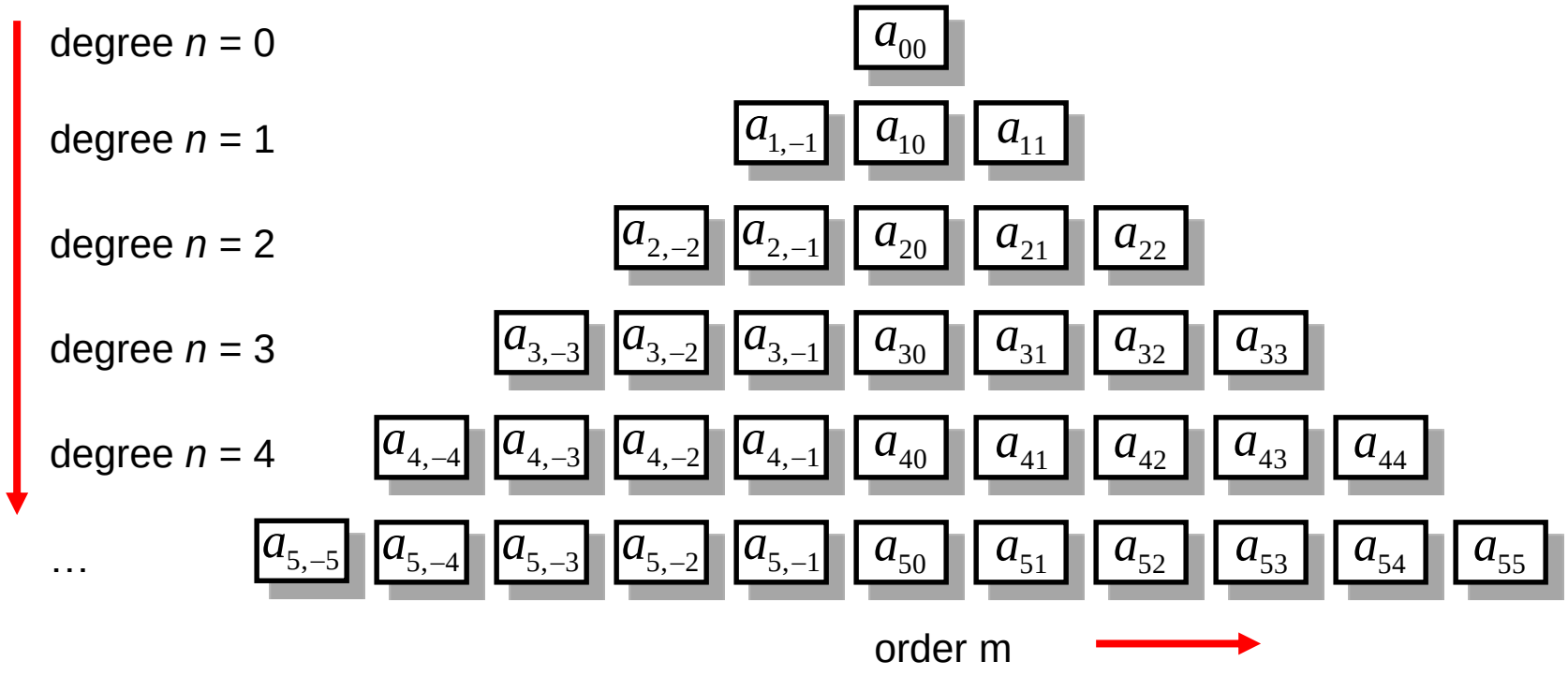
Physical interpretation of the coefficients

Coefficient triangle

Approximation of a function on the sphere

$$f(\lambda,\vartheta)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}Y_{nm}(\lambda,\vartheta)$$

Arrangement of coefficients in a triangle



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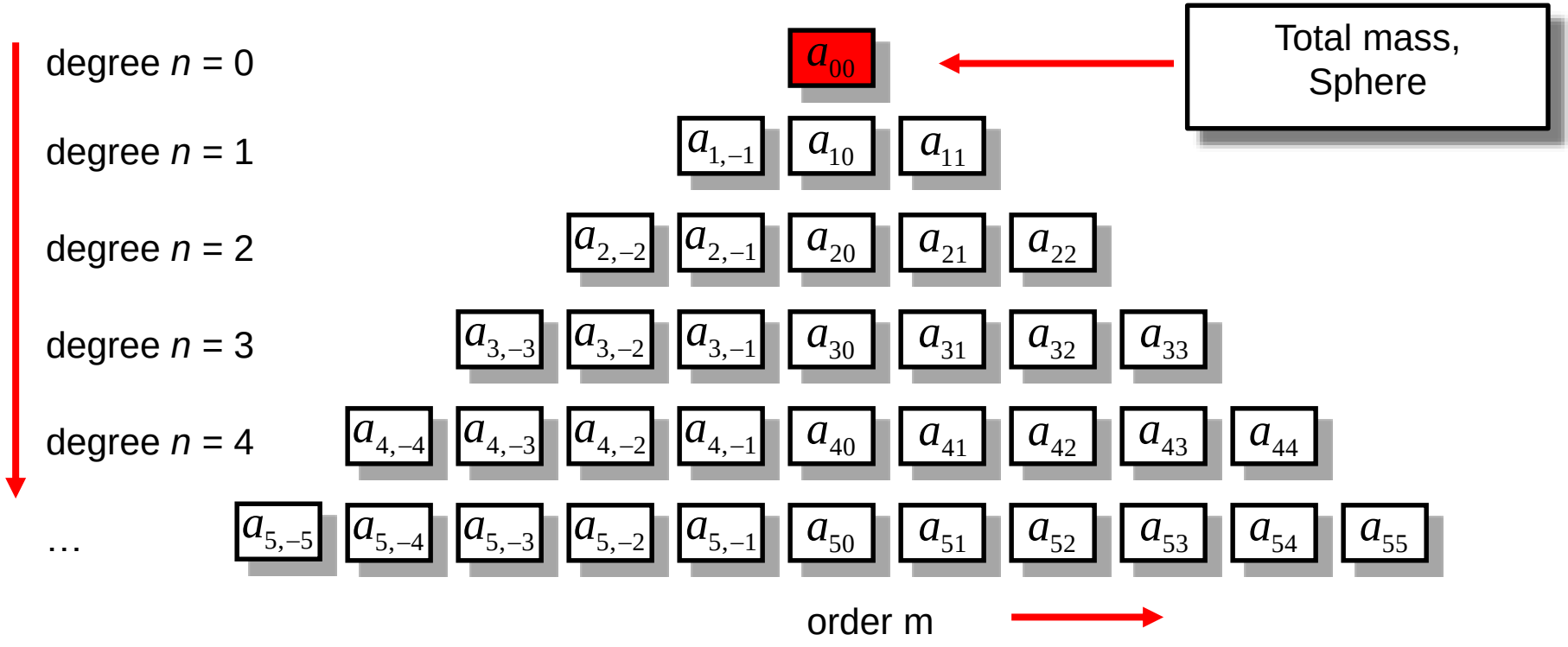

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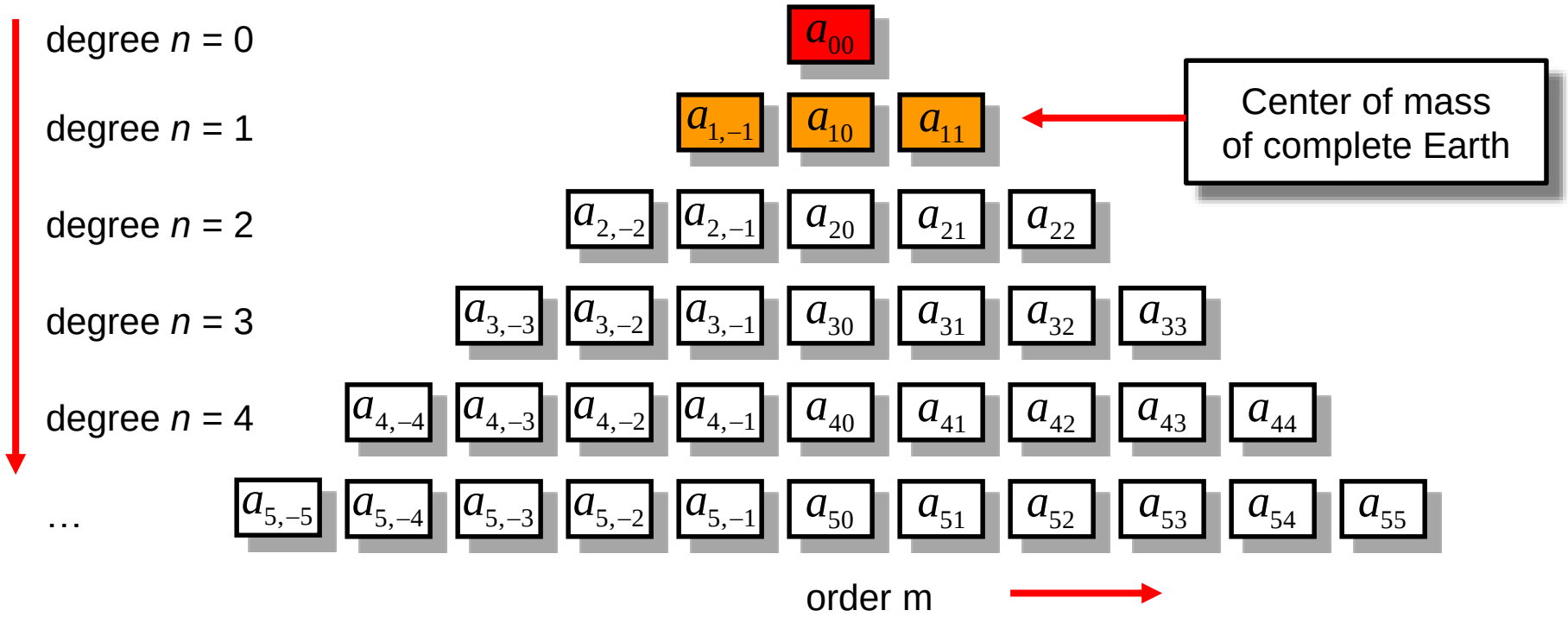
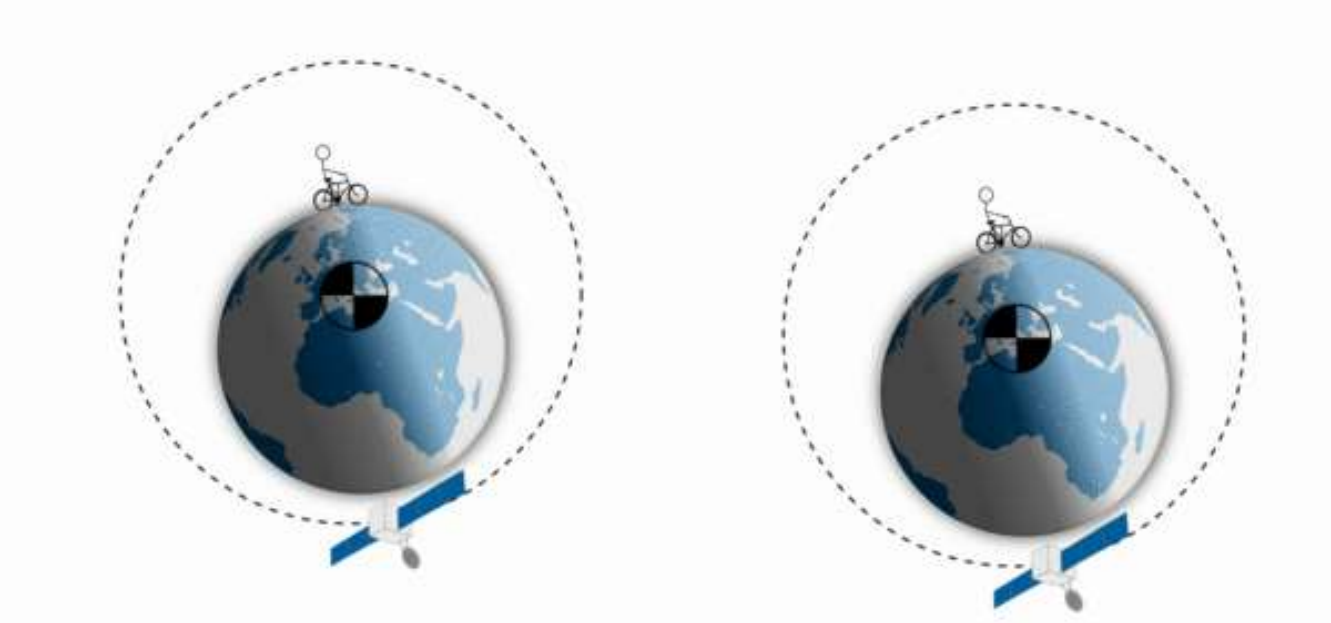


Geocenter motion

Earth fixed

GRACE

R. Rietbroek



To analyize data in an Earth fixed system, external time series of degree 1 coefficients are needed.

GRACE orbits around the center of total mass (Fluids (bycycle) + Solid Earth)
⇒ Degree 1 coefficients are always zero

e.g.:
www.igg.uni-bonn.de/apmg/index.php?id=geozentrum

Coefficient triangle

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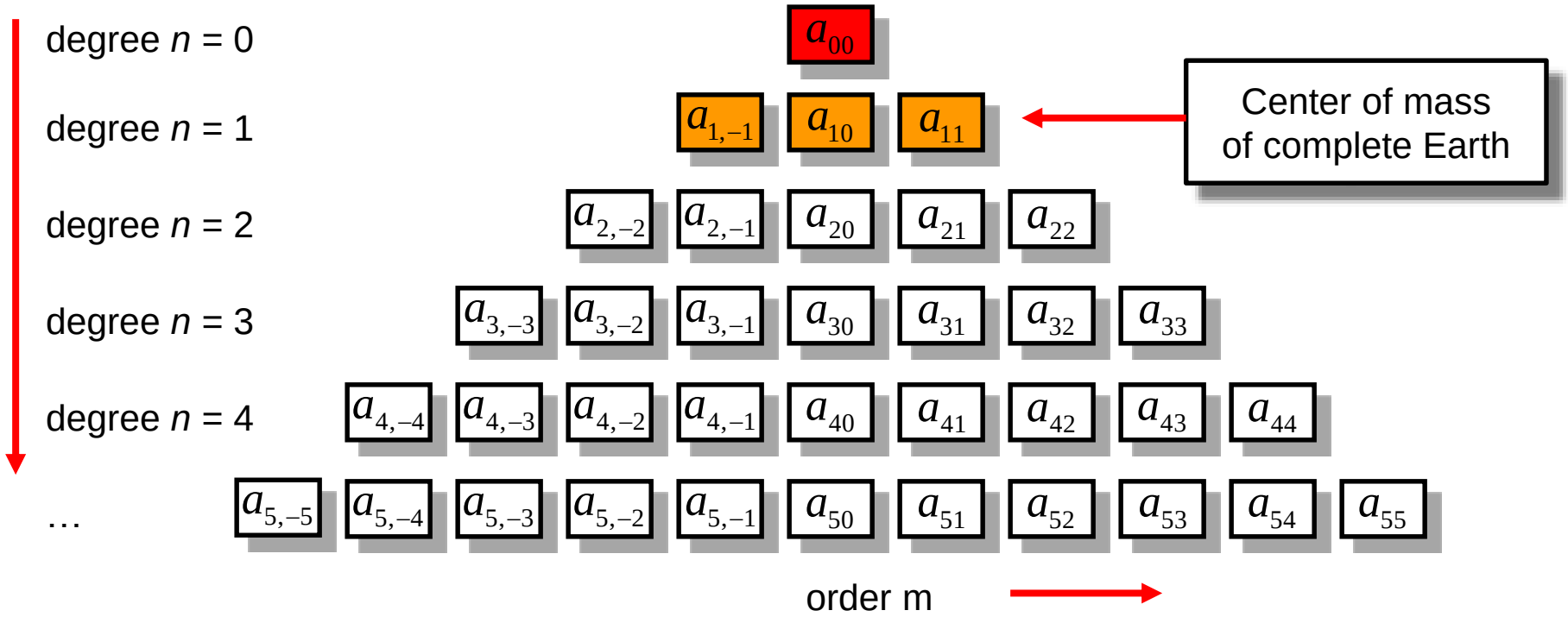

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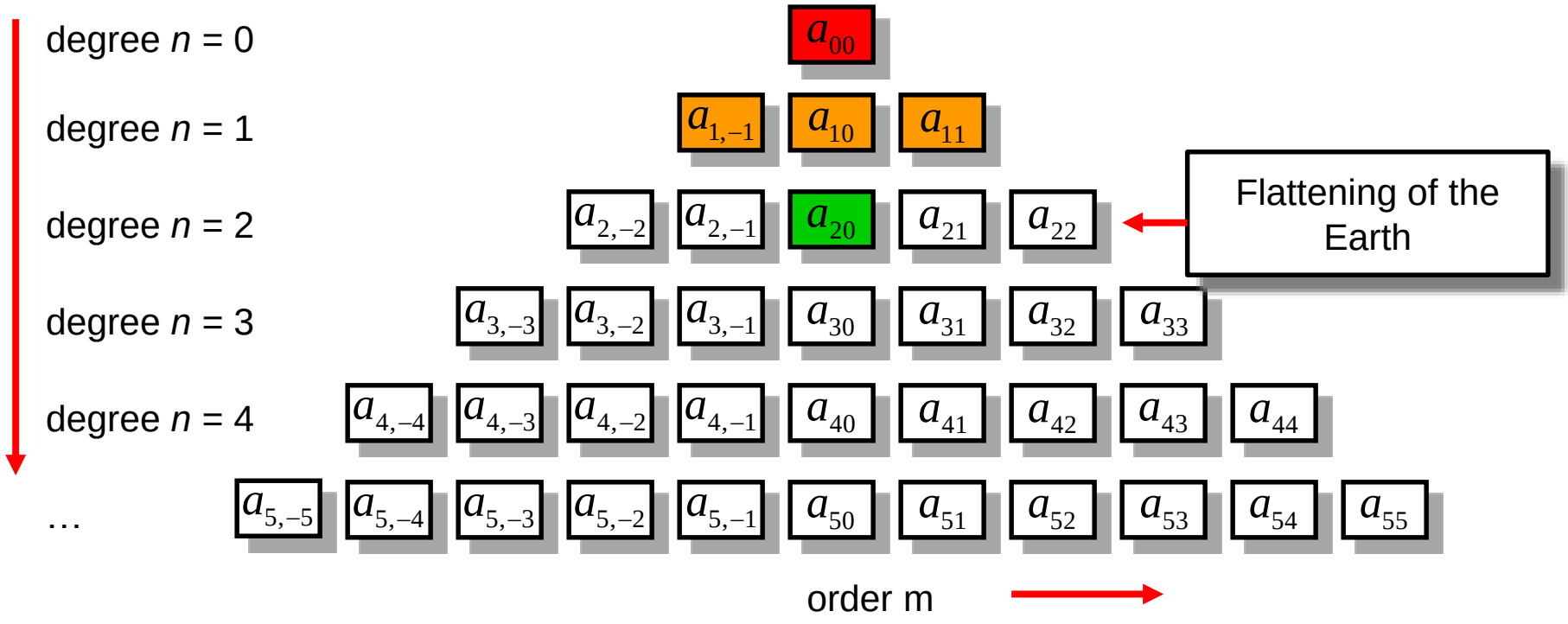


Signal content

Signal content

All masses causes gravity
⇒ GRACE cannot separate the different signal sources

- Hydrology
- Ice sheets
- Glaciers
- Permanent frost
- Ocean tides
- Ocean pole tides
- Barotropic ocean circulation
- Sea level rise
- Atmospheric tides (S1, S2)
- Atmospheric mass redistribution
- Solid Earth tides
- Rotational deformation (pole tides)
- Glacial isostatic adjustment
- Loading deformation
- Degree 1 mass redistribution
- Earthquakes

Ice

Ocean

Atmosphere

Solid Earth

Signal content

All masses causes gravity
⇒ GRACE cannot separate the different signal sources

- Hydrology
- Ice sheets
- Glaciers
- Permanent frost
- ~~Ocean tides~~
- ~~Ocean pole tides~~
- Barotropic ocean circulation
- Sea level rise
- ~~Atmospheric tides (S1, S2)~~
- ~~Atmospheric mass redistribution~~
- ~~Solid Earth tides~~
- ~~Rotational deformation (pole tides)~~
- Glacial isostatic adjustment
- Loading deformation
- Degree 1 mass redistribution
- Earthquakes

Some of the signals are not included in the GRACE solutions, but directly reduced by models

Evaluation of GRACE solutions
Step by Step

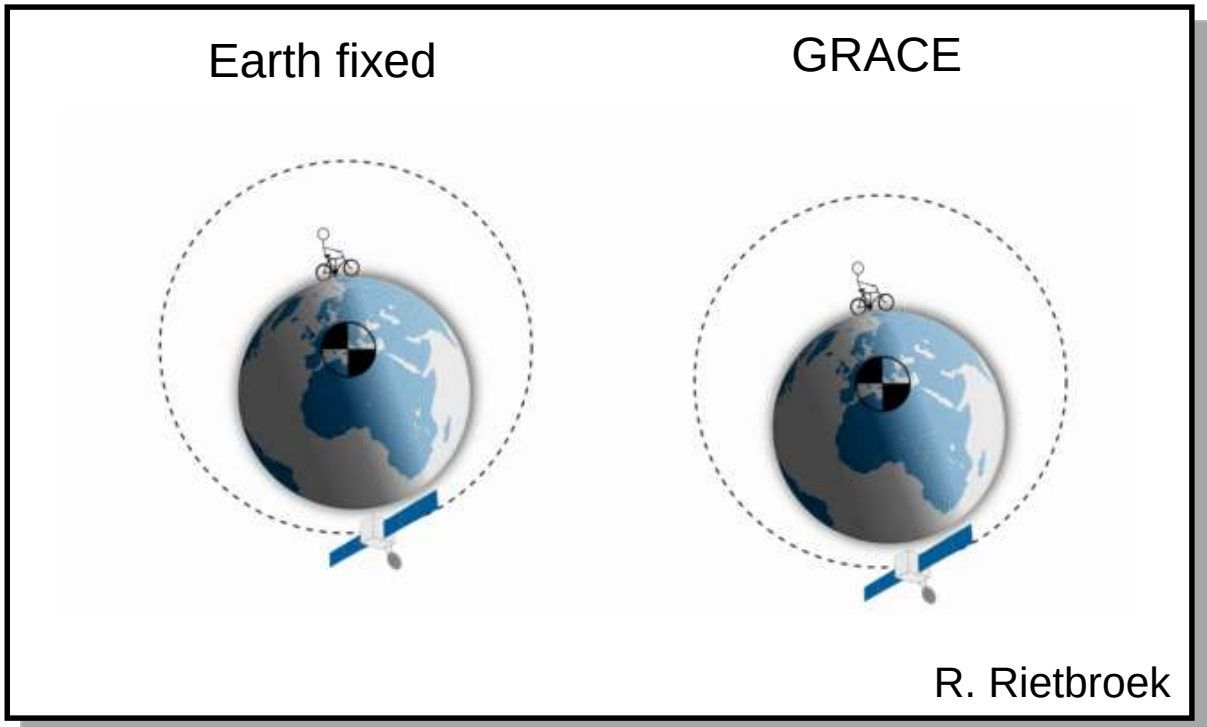
Step by Step

1. Download the spherical harmonics coefficients

```
product_type            gravity_field
modelname               ITG-Grace03
comment                 static field from 2002-08 to 2007-04 of GRACE dat
earth_gravity_constant 3.986004415e+14
radius                  6378136.6
max_degree              180
key n m        C                 S                 sigma C           si
end_of_head ===================================================================
gfc 0 0  1.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000
gfc 1 0  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000
gfc 1 1  0.000000000000e+00  0.000000000000e+00 0.000000000000e+00 0.000
gfc 2 0 -4.841692718699e-04  0.000000000000e+00 6.469883774458e-13 0.000
gfc 2 1 -2.654790999243e-10  1.475393314283e-09 6.108979511966e-13 6.355
gfc 2 2  2.439383367978e-06 -1.400273635220e-06 6.254221806143e-13 6.423
gfc 3 0  9.571610348416e-07  0.000000000000e+00 4.908157850872e-13 0.000
gfc 3 1  2.030461736678e-06  2.482003394707e-07 4.904543816334e-13 5.118
gfc 3 2  9.047877724984e-07 -6.190053685183e-07 5.459595906001e-13 5.482
gfc 3 3  7.213217237276e-07  1.414349090196e-06 5.163836126113e-13 5.163
gfc 4 0  5.399657665980e-07  0.000000000000e+00 3.758481731782e-13 0.000
gfc 4 1 -5.361573220519e-07 -4.735672404588e-07 3.874699557956e-13 3.973
gfc 4 2  3.505015650151e-07  6.624798955603e-07 4.501829959710e-13 4.398
gfc 4 3  9.908565738322e-07 -2.009566568843e-07 4.776067657084e-13 4.766
gfc 4 4 -1.885196275153e-07  3.088038091544e-07 4.556511148108e-13 4.565
gfc 5 0  6.867029195170e-08  0.000000000000e+00 2.444626602968e-13 0.000
gfc 5 1 -6.292117216708e-08 -9.436975416042e-08 2.510438328896e-13 2.640
```

Step by Step

1. Download the spherical harmonics coefficients
2. Insert external coefficients of degree 1



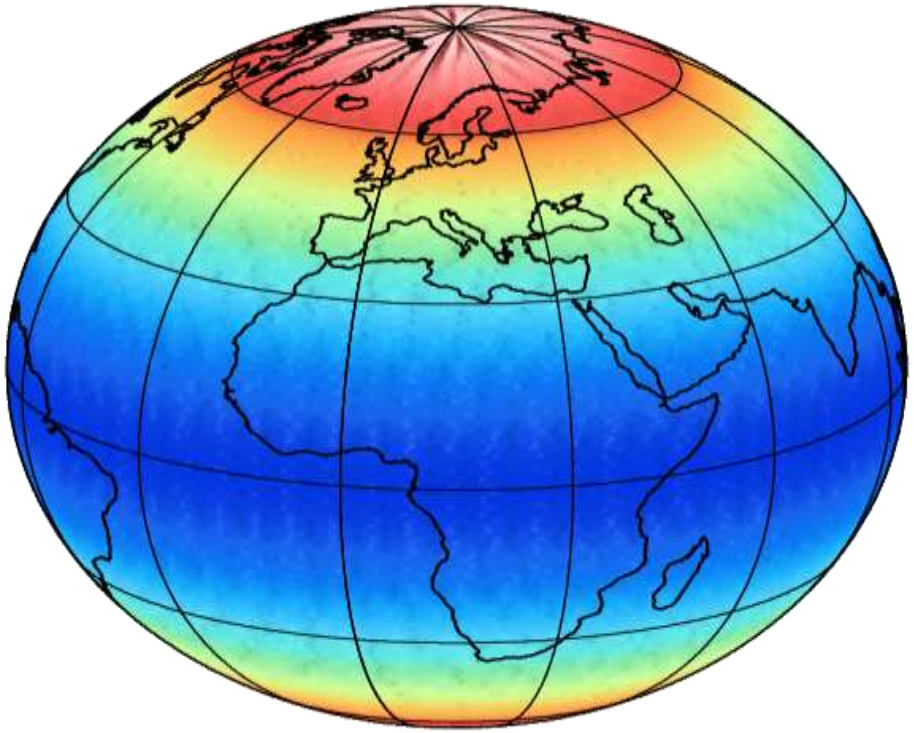
Step by Step

1. Download the spherical harmonics coefficients

2. Insert external coefficients of degree 1

3. Remove static part of the gravity field
   (temporal mean of time series
   or other static field e.g. GOCO05s)

$$\Delta a_{nm} = a_{nm} - a^{0}_{nm}$$



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$$\Delta a_{nm} = a_{nm} - a^0_{nm}$$

4. Remove unwanted signals, e.g. GIA
5. Filter the coefficients (e.g. Gaussian filter)

$$\Delta \bar{a}_{nm} = w_n \Delta a_{nm}$$

Filter coefficients

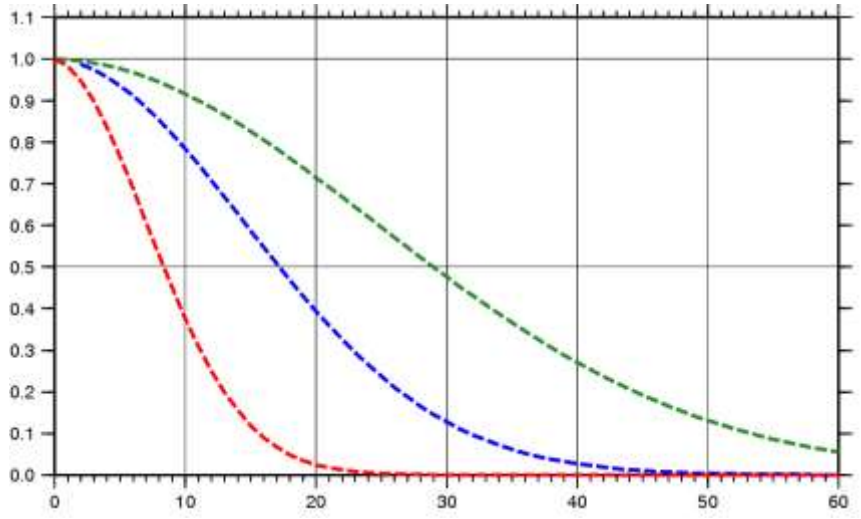
$$w_0 = 1$$

$$w_1 = \frac{1+e^{-2b}}{1-e^{-2b}} - \frac{1}{b}$$

$$w_n = -\frac{2n-1}{b} w_{n-1} + w_{n-2}$$

with $b = \frac{\ln(2)}{1-\cos(\frac{d}{R})}$

with *d*: filter radius



Step by Step

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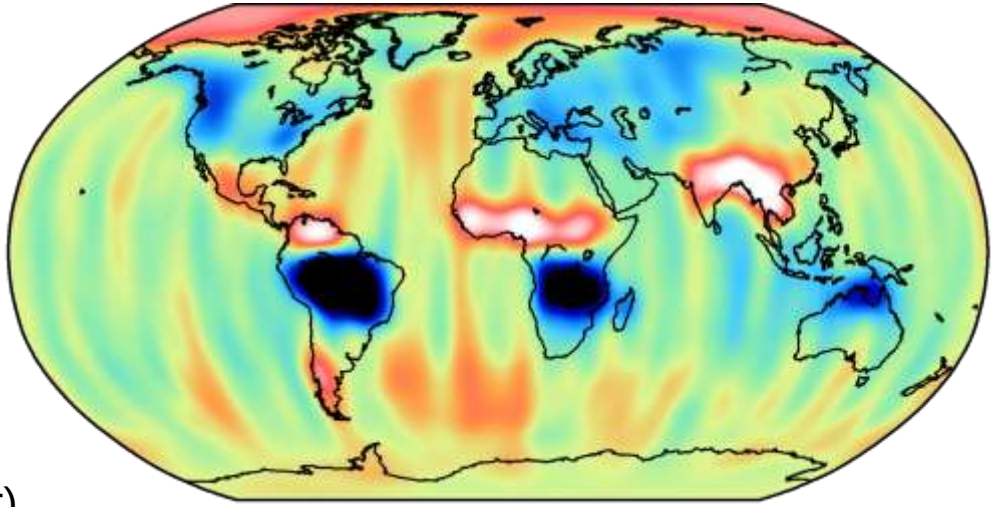

4. Remove unwanted signals, e.g. GIA
5. Filter the coefficients (e.g. Gaussian filter)

$$\Delta \bar{a}_{nm} = w_n \Delta a_{nm}$$

6. Compute Total water storage changes (TWS) at each grid point (λ_i, ϑ_i)

$$\Delta TWS(\lambda,\vartheta) = \frac{\rho_e R}{3} \sum_{n=0}^{\infty} \frac{2n+1}{1+k_n^{'}} \sum_{m=-n}^{n} \Delta a_{nm} Y_{nm}(\lambda,\vartheta)$$